i-Ready Classroom Mathematics Research Base

Grades K–8

The mission of Curriculum Associates is to make classrooms better places for teachers and students.

Contents

Introduction

i-Ready Classroom Mathematics is a comprehensive core mathematics program that flexibly embeds opportunities for students to take ownership of their learning. It empowers students with the conceptual understanding and fluency necessary to promote their growth in mathematics. The program equally emphasizes three aspects of rigor with a balanced approach that builds students' conceptual understanding of mathematics, develops their procedural fluency around them, and provides opportunities for students to demonstrate their learning through application to new situations.

Moreover, *i-Ready Classroom Mathematics* is intentionally designed and written to support teachers in identifying where students are in their mathematical understanding in order to accelerate their progress toward grade level and beyond. The ongoing progress monitoring allows teachers to see what their students know and still need to learn. Equipped with this information, teachers flexibly use the *i-Ready Classroom Mathematics* program to provide data-driven and research-informed instruction that is differentiated to meet students where they are to propel them forward.

The following fundamental principles guide the work of *i-Ready Classroom Mathematics*:

- **Put Students at the Heart of Learning**: Deepen students' understanding of mathematics and encourage them to participate actively in math class by responding to their unique learning styles.
- **Focus on High-Impact Teaching Strategies**: Maximize student growth by using a student-centered approach that incorporates engaging routines, activities, and discourse to help build conceptual understanding, develop procedural fluency, and support mathematical application.
- **Turn Data into Action**: Teachers use data from both formal and informal assessments to inform instructional decisions.
- **Support Teachers Every Step of the Way**: The program is designed to provide teachers with ample support and resources so they can meet their students' learning needs.

The following pages walk through the research base upon which the *i-Ready Classroom Mathematics* program was built and how it makes mathematics accessible to all students.

Authors and Advisors

i-Ready Classroom Mathematics provides evidence-based instruction informed by program authors, advisors, and a panel of expert educators representing teachers across the country. Guidance from our authors and advisors ensured the design of a rigorous program that provides students with opportunities to practice and apply new learning purposefully and meaningfully while being manageable for teachers to implement.

Dr. Mark Ellis, Ph.D.

Awards and Key Positions

- Board of Directors, Executive Committee, National Council of Teachers of Mathematics (NCTM)
- Department Chair and Professor, Education, California State University (CSU) Fullerton
- Distinguished Faculty, College of Education, CSU Fullerton
- Certification Council, National Board for Professional Teaching Standards (NBPTS)
- National Board Certified Teacher
- Research to Practice Award, Mathematics Teaching in the Middle School

Publications and Advisory Focus

- Coauthor of *Reimagining the Mathematics Classroom: Creating and Sustaining Productive Learning Environments, K–6*
- Equitable and inclusive practices in mathematics education
- Culturally responsive mathematics teaching and learning
- Preparation of teachers of mathematics
- History of school mathematics in the United States

Gladis Kersaint, Ph.D.

Awards and Key Positions

- Professor of Mathematics Education and Vice Provost for Strategic Initiatives, University of Connecticut
- Dean, Neag School of Education, University of Connecticut, 2016–2021
- Professor of Mathematics Education, University of South Florida, 1998–2016
- Board of Directors, NCTM, 2012–2015
- Board of Directors, Association of Mathematics Teachers Educators (AMTE), 2008–2011
- Mathematics teacher, Miami-Dade County Public Schools

Publications and Advisory Focus

- Coauthor of *Teaching Mathematics to English Language Learners* and *Mathematical Literacy: Helping Students Make Meaning in the Middle Grades*
- Equity and discourse in mathematics education
- Mathematics teaching and learning
- Preparation of teachers of mathematics

Grace Kelemanik, M.S.T.

Awards and Key Positions

- Cofounder, Fostering Math Practices
- National consultant supporting teachers, coaches, and school leaders
- Education Development Center, project director
- Boston Teacher Residency Program, teacher educator
- Urban Grades 6–12 mathematics teacher and leader

Publications and Advisory Focus

- Coauthor of *Routines for Reasoning: Fostering the Mathematical Practices in All Students* and *Teaching for Thinking: Fostering Mathematical Practices Through Reasoning Routines*
- Teaching with instructional routines and implementing high-leverage pedagogical strategies
- Urban education
- Mathematics instruction for special populations
- Teacher development

Amy Lucenta, M.Ed.

Awards and Key Positions

- Cofounder, Fostering Math Practices
- National consultant supporting teachers, coaches, and school leaders
- Mathematics teacher and leader (Grades K–12)

Publications and Advisory Focus

- Coauthor of *Routines for Reasoning: Fostering the Mathematical Practices in All Students* and *Teaching for Thinking: Fostering Mathematical Practices Through Reasoning Routines*
- Teaching with instructional routines and implementing equitable teaching practices
- Integrating mathematical practices into instruction
- Engaging and supporting all learners

Advisors

Michael Flynn

Advisory Focus: Grades K–2 Enrichment

Mike Flynn, director of the mathematics leadership programs at Mount Holyoke College, Massachusetts, is an active leader in mathematics education. He is the author of numerous publications on mathematics teaching, including *Beyond Answers: Exploring Mathematical Practices with Young Children.*

Bob Dolan, Ph.D.

Advisory Focus: Universal Design for Learning

Bob Dolan, Ph.D., Diverse Learners Consulting, brings 30+ years of experience in neuroscience, learning science, instructional design, assessment, and software development. He is an expert on applying the Universal Design for Learning (UDL) to instruction and assessment in both publishing and research environments.

Dr. Sharroky Hollie

Advisory Focus: Cultural Responsiveness

Professor Sharroky Hollie provides professional development to thousands of educators in the area of cultural responsiveness. Since 2000, Dr. Hollie has trained more than 150,000 educators and worked in nearly 2,000 classrooms. He has been a classroom teacher, a central office professional development coordinator in the Los Angeles Unified School District, a school founder and administrator, and a university professor in teacher education at Cal State University. He wrote *Culturally and Linquistically Responsive Teaching and Learning: Classroom Practices for Student Success*, published in 2011, and *Strategies for Culturally and Linguistically Responsive Teaching and Learning* in 2015.

English Learners Success Forum (ELSF)

Advisory Focus: Academic Language and Discourse

The English Learners Success Forum is a collaboration of researchers, teachers, state and district leaders, content creators, and education funders who are dedicated to improving the quality and accessibility of instructional materials for English Learners (ELs). ELSF's experts provide guidance to curriculum developers in addressing the linguistic and cultural assets and needs of ELs with the goal of providing them full access to grade-level content and quality learning.

Sarah Bent

Advisory Focus: Grades 3–5 Enrichment

Sarah Bent, assistant director of the mathematics leadership programs at Mount Holyoke College, Massachusetts, supports professional learning programs for Grades K–8 mathematics teachers around the country. Sarah is a former Grades 3 and 4 teacher.

Cathery Yeh

Advisory Focus: Equity and Inclusion

Cathery Yeh is an assistant professor at Chapman University. She has been in education for more than 20 years, beginning her tenure in dual-language classrooms in Los Angeles and Orange County, CA and abroad. At Chapman University, Cathery teaches mathematics methods with a focus on teaching practices that challenge deficit-based thinking and draw on student strengths. She also developed the community math learning program and has authored many journals and books, including coauthoring *Reimagining the Mathematics Classroom*. Cathery is the 2022 Early Career Award recipient from the AMTE and a member of the NCTM Board of Directors.

Harold Asturias

Advisory Focus: Equity and ELs

Harold Asturias is the director of the Center for Mathematics Excellence and Equity at the University of California, Berkeley. He provides professional development in the areas of standards and assessment in mathematics for large urban districts and smaller rural districts. His current focus is supporting Grades K–12 teachers with students from underrepresented and underserved groups, connecting mathematics and English language development.

The Council of the Great City Schools

Advisory Focus: ELs

The Council of the Great City Schools brings together 76 of the nation's largest public school systems in a coalition dedicated to the improvement of education for children in the inner cities. Mathematics and ELs experts from member districts provided feedback to Curriculum Associates and other publishers to meet the criteria for instructional materials set forth in *A Framework for Re-envisioning Mathematics Instruction for English Language Learners* (The Council of the Great City Schools, 2019).

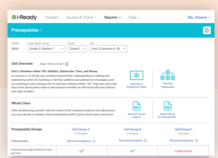
Program Components

Assessment

Diagnose, Screen, and Monitor

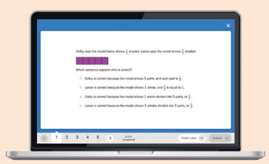
Diagnostic E/S

This adaptive assessment provides teachers with actionable insight into student needs and charts a personalized learning path for each student.



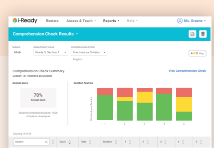
Prerequisites Report

Address unfinished learning during small group or whole class instruction, depending on the needs of the class.



Comprehension Checks E/S

Save time with auto-graded assessments comparable to the Lesson Quizzes and Mid-Unit and Unit Assessments.



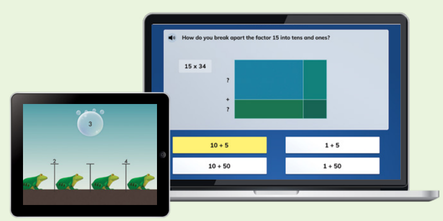
Comprehension Check Reports

In-depth analysis of student understanding, including the rationale for typical incorrect responses, helps identify student misconceptions.

Whole Class Instruction and Practice

Student Components E/S

Students take ownership of their learning with these engaging print and digital resources.



Student Worktexts, Fluency and Skills Practice Book, Assessment Practice Book, Student Digital Experience (including digital practice and virtual manipulatives), and Manipulative Kits*

*Available for purchase at Hand2Mind.com/Curriculum-Associates

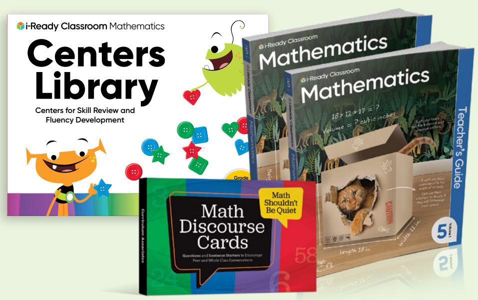
E/S Available in English and Spanish

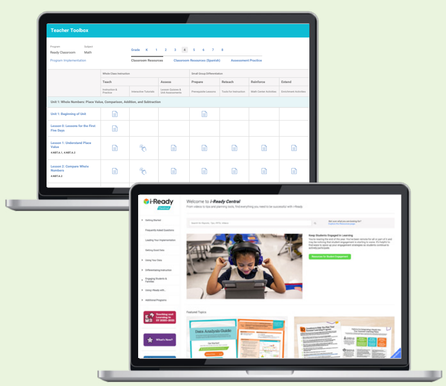
***i-Ready Classroom Matemáticas* also available!**

Whole Class Instruction and Practice

Teacher Components E/S

Make math accessible and build students' confidence with these high-quality instructional materials.



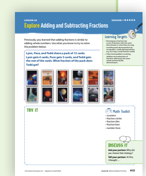


Teacher's Guides, Discourse Cards, Centers Library (Grades K–1), *i-Ready Success Central*, and Teacher Digital Experience (including reports, Teacher Toolbox, assignable practice, and program implementation resources and support)

Small Group Differentiation

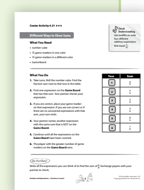
Prerequisite Lessons E/S

In-depth instruction that reviews prerequisite concepts



Math Center Activities and Enrichment Activities E/S

Targeted resources to help students grow and succeed



Tools for Instruction PDFs E/S

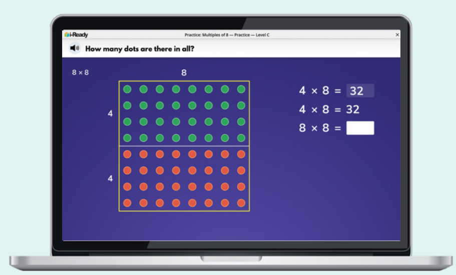
Teacher-led activities for small group instruction addressing prerequisite or on-grade level skills

Add On

Personalized Learning and Intervention

i-Ready Personalized Instruction E/S

Driven by results from the *i-Ready Diagnostic*, these interactive lessons provide instruction tailored to each student's needs.



Evidence Base

Effective Mathematics Teaching Practices Explored

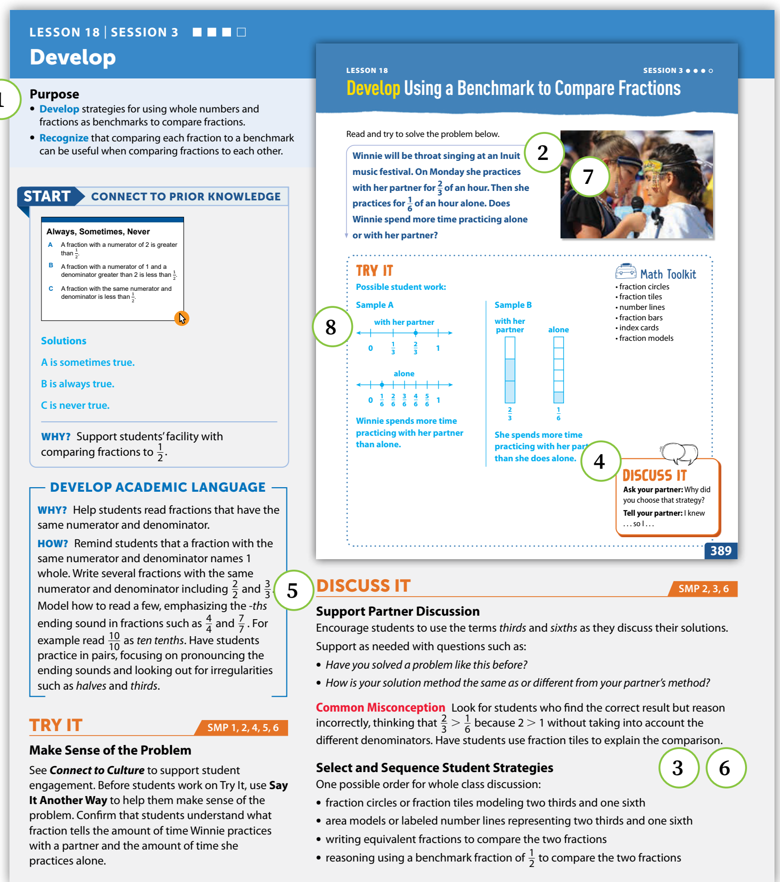
i-Ready Classroom Mathematics is a student-centered, core mathematics program designed to prepare all students to succeed with grade-level content. Utilizing flexible, equitable, accessible instruction, *i-Ready Classroom Mathematics* embeds and integrates best practices for teaching mathematics into daily instruction to develop the habits of mind of mathematically proficient students. A student's experience in learning mathematics has a substantial influence on their cognitive processing, which has the potential to shape their impression of mathematics and their view of themselves as mathematicians (Aguirre et al., 2013). To provide every student with the opportunity to reach their full mathematical potential, the program intentionally helps students make connections and deepen their understanding of mathematical concepts, apply their knowledge in novel situations, demonstrate mastery, and experience the wonder, joy, and beauty of mathematics (NCTM, 2020a).

i-Ready Classroom Mathematics is grounded in the following Effective Mathematics Teaching Practices as defined by the NCTM (NCTM, 2014):

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

This section explores each of these Effective Mathematics Teaching Practices and explains how *i-Ready Classroom Mathematics* purposefully embeds them into the program.

Instruction Grounded in the NCTM's Effective Mathematics Teaching Practices



Purposeful Instruction: Lessons in i-Ready Classroom Mathematics *incorporate NCTM's Effective Mathematics Teaching Practices into instruction.*

1. Establish Mathematics Goals to Focus Learning

"Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions" (NCTM, 2014).

Goals for learning mathematics set the stage for the instruction that is to follow (Wiliam, 2011). They describe the mathematical practices and processes students are to learn in order to be successful mathematicians (NCTM, 2020a). In other words, mathematical goals are the guidelines for student success and describe mathematical concepts, ideas, or methods students are expected to understand as a result of instruction (Hiebert et al., 2007). Mathematical goals also demystify the learning process for teachers and students. As a result of clearly established goals, both teachers and students should be able to identify the mathematical concepts that will be learned throughout the course of study, their relevance, and their connection to previously taught material as they gain an appreciation for what will come next in the learning process (NCTM, 2020a).

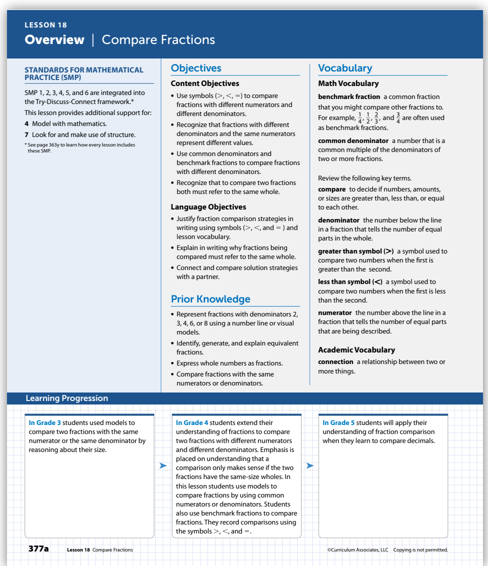
Clearly articulated goals lead to high expectations for every student within a class (NCTM, 2020a; Marzano, 2003; McTighe & Wiggins, 2013). Classrooms in which students understand the learning goals and expectations for their work perform at higher levels than in classrooms where the expectations are not clear (Haystead & Marzano, 2009; Hattie, 2009). As a result, students who experience high expectations see themselves as confident and competent doers of mathematics (Turner et al., 2013).

Learning goals and instructional supports in *i-Ready Classroom Mathematics* emphasize content standards, language development, Standards for Mathematical Practice, and daily language objectives. For example:

- Learning Targets are in every lesson in the Student Worktext and the Teacher's Guide.
- Content Objectives and Language Objectives are in every lesson in the Teacher's Guide.
- Purpose Statements are in every lesson in the Teacher's Guide.

In all grades, students have opportunities to revisit and reflect on skills learned and goals met in each unit.

Content Standards Connect to Mathematical Practice and Language Objectives


LESSON 18

Overview | Compare Fractions

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect framework.*

This lesson provides additional support for:

4 Model with mathematics.

7 Look for and make use of structure.

* See page 363y to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Use symbols (>, <, =) to compare fractions with different numerators and different denominators.
- Recognize that fractions with different denominators and the same numerators represent different values.
- Use common denominators and benchmark fractions to compare fractions with different denominators.
- Recognize that to compare two fractions both must refer to the same whole.

Language Objectives

- Justify fraction comparison strategies in writing using symbols (>, <, and =) and lesson vocabulary.
- Explain in writing why fractions being compared must refer to the same whole.
- Connect and compare solution strategies with a partner.

Prior Knowledge

- Represent fractions with denominators 2, 3, 4, 6, or 8 using a number line or visual models.
- Identify, generate, and explain equivalent fractions.
- Express whole numbers as fractions.
- Compare fractions with the same numerators or denominators.

Vocabulary

Math Vocabulary

benchmark fraction a common fraction that you might compare other fractions to. For example, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ are often used as benchmark fractions.

common denominator a number that is a common multiple of the denominators of two or more fractions.

Review the following key terms.

compare to decide if numbers, amounts, or sizes are greater than, less than, or equal to each other.

denominator the number below the line in a fraction that tells the number of equal parts in the whole.

greater than symbol (>) a symbol used to compare two numbers when the first is greater than the second.

less than symbol (<) a symbol used to compare two numbers when the first is less than the second.

numerator the number above the line in a fraction that tells the number of equal parts that are being described.

Academic Vocabulary

connection a relationship between two or more things.

Learning Progression

In Grade 3 students used models to compare two fractions with the same numerator or the same denominator by reasoning about their size.

In Grade 4 students extend their understanding of fractions to compare two fractions with different numerators and different denominators. Emphasis is placed on understanding that a comparison only makes sense if the two fractions have the same-size wholes. In this lesson students use models to compare fractions by using common numerators or denominators. Students also use benchmark fractions to compare fractions. They record comparisons using the symbols >, <, and =.

In Grade 5 students will apply their understanding of fraction comparison when they learn to compare decimals.

377a Lesson 18 Compare Fractions ©Curriculum Associates, LLC Copying is not permitted.

Every lesson contains Learning Targets, Content Objectives, and Language Objectives.

2. Implement Tasks That Promote Reasoning and Problem Solving

"Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies" (NCTM, 2014).

To ensure the mathematical success of all students, teachers must regularly select and implement tasks that promote reasoning and problem solving. Tasks should encourage reasoning and access to learning through using different representations and tools, and they should promote problem solving through considering various strategies to reach a solution (NCTM, 2020a).

Learning mathematics through tasks and problem solving motivates students and helps them construct new knowledge. Not all tasks, however, provide the same thinking and learning opportunities for students (Hiebert et al., 1997; Stein et al., 2009). Student learning is greatest in classrooms that encourage high levels of student thinking compared with classrooms where tasks are procedural in nature (Boaler & Staples, 2008; Hiebert & Wearne, 1993; Stein & Lane, 1996). Tasks should encourage students to make sense of concepts and approach solving math problems with a focus on multiple entry points (Boaler & Staples, 2008; Small, 2017). They should also be culturally relevant and draw on students' funds of knowledge (NCTM, 2020a).

i-Ready Classroom Mathematics provides students opportunities to engage in higher-order thinking skills. Throughout the program, engaging activities are used to introduce students to new concepts while piquing their interest and activating prior knowledge. By leveraging previously learned concepts and strategies, students have multiple entry points to apply previously secured understandings to new content and materials. For example, in the Try It part of the Try–Discuss–Connect instructional framework, students collaborate to make sense of the problem using a language routine, then they solve it on their own and support their thinking. The multiple entry points in the Try It tasks allow students to choose the tools or strategies they wish to use to solve the problem.

Students also have the opportunity to appreciate varied solution strategies as they share their thinking with partners and compare their strategies. Teachers identify and select strategies to use and invite students to share their thinking and solutions in a sequence that builds conceptual understanding. The process of involving both peer-to-peer and whole class discourse provides engagement opportunities for students to learn from one another, practice the language of mathematics, and build their mathematical confidence.

A Powerful Framework for Promoting Problem Solving and Reasoning

TRY IT	DISCUSS IT	CONNECT IT
Students make sense of the problem.	Students share their thinking with a partner.	Students make connections and reflect on what they have learned.
Students solve and support their thinking.	Students compare their strategies.	Students apply their thinking to a new problem.

The Try–Discuss–Connect instructional framework puts students at the center of learning.

3. Use and Connect Mathematical Representations

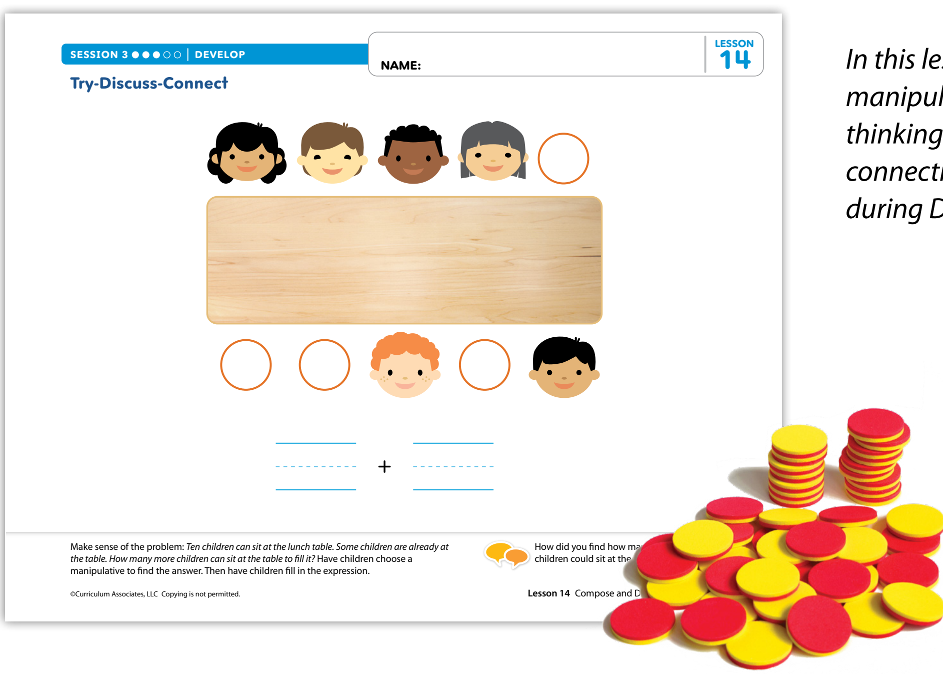
"Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving" (NCTM, 2014).

Mathematical representations are important because they help students understand concepts and procedures, make sense of problems, and engage in mathematical discourse (Arcavi, 2003; Stylianou & Silver, 2004). They also help students access the abstract nature of mathematics (National Research Council, 2001). In fact, students' depth of understanding is related to the strength of connections they make among mathematical representations they have internalized (Pape & Tchoshanov, 2001; Webb et al., 2008).

Examples of mathematical representations appear in the Model It and Picture It parts of the lesson and increase in complexity across the program. These parts of the lesson support students as they consider the relationships among quantities when they sketch their diagrams or make tables and graphs. When students learn to use visuals to convey, discuss, and make connections among mathematical ideas in multiple forms, they demonstrate deeper mathematical understanding and enhanced problem-solving abilities (Fuson et al., 2005; Lesh et al., 1987). Mathematical representations provide access and allow more students who learn in a variety of ways to participate meaningfully in the mathematical discourse in the classroom. Visual supports also enable ELs to make connections between concepts and language (Fuson & Murata, 2007).

Students engage with hands-on activities and create mathematical representations throughout the *i-Ready Classroom Mathematics* program to help communicate their ideas and thinking, make connections, see relationships, and visualize how concepts build on each other. During the Discuss It portion of the framework, teachers can use the Compare and Connect routine so that students see the relationship between different mathematical representations. Oftentimes, the Model It and Picture It section in a session helps students further understand the relationship between concrete, representational, and abstract models. By building upon their existing knowledge and making connections across multiple strategies and representations, students deepen their understanding, build flexibility in their thinking, gain the ability to apply their knowledge to novel problems, and better retain what they have learned.

Students Use a Variety of Mathematical Representations to Solve Problems



In this lesson, students use manipulatives to represent their thinking during Try It and then make connections between strategies during Discuss It and Connect It.

4. Facilitate Meaningful Mathematical Discourse

"Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (NCTM, 2014).

Effective mathematics teaching engages students in discourse to advance mathematical learning and thinking through the purposeful use of language. Mathematical discourse includes thoughtful exchange of ideas through classroom discussion as well as through other forms of verbal, visual, and written communication. The discourse in the mathematics classroom gives students opportunities to share ideas and clarify understandings, construct arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from others' perspective (NCTM, 2014). Students who learn to articulate and justify their own mathematical ideas, reason through their own and others' mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical to their future success in mathematics and related fields (Carpenter et al., 2003).

Discourse that focuses on tasks that promote reasoning and problem solving is a primary mechanism for developing conceptual understanding and meaningful learning of mathematics (Michaels et al., 2008). Establishing classrooms rich with equitable opportunities to participate in mathematical discourse supports students' positive mathematical identities, establishes shared mathematical authority in the classroom, and aids in equalizing student status (Wood et al., 2019). In order for mathematical discourse to occur regularly within classrooms, students are encouraged to see the value in attending to the mathematical ideas of their peers (Kelemanik et al., 2016).

i-Ready Classroom Mathematics strategically embeds language routines and teacher moves into the Try–Discuss–Connect framework to provide students with time and space to develop their own ideas and make sense of and analyze others' ideas. Furthermore, the language routines support students by providing a familiar way of focusing on language and its important role in communicating and processing mathematical understandings and ideas.

Integrate Language and Mathematics

TRY IT

Language Routines
- Three Reads
- Co-Craft Questions
- Notice and Wonder
- Say It Another Way

Teacher Moves
- Turn and Talk
- Individual Think Time

DISCUSS IT

Language Routines
- Collect and Display
- Compare and Connect

Teacher Moves
- Turn and Talk
- Individual Think Time
- Four Rs

Conversation Tips

CONNECT IT

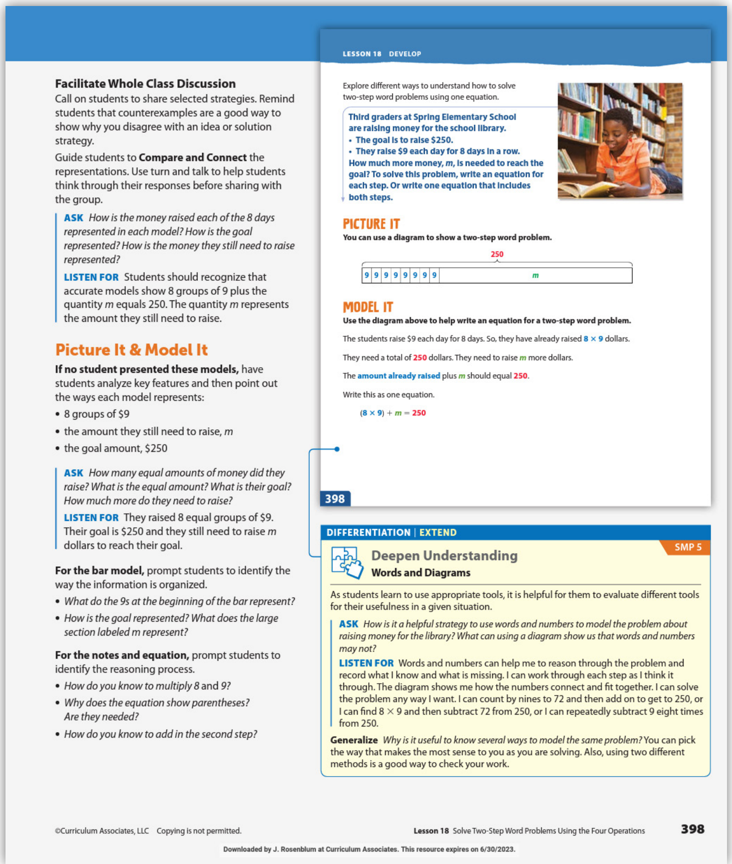
Language Routines
- Collect and Display
- Compare and Connect

Teacher Moves
- Turn and Talk
- Individual Think Time
- Four Rs

i-Ready Classroom Mathematics *provides supports for language development and discourse including language routines, teacher moves, and conversation tips.*

Research-based language routines are used consistently throughout session activities, and support features help students engage in deep thinking and learning and use of language. These routines include: Three Reads, Notice and Wonder, Say It Another Way, Co-craft Questions, Compare and Connect, Collect and Display, Act It Out, Co-constructed Word Banks, Stronger and Clearer Each Time, and Academic Vocabulary Routine. The teacher moves support students' capacity to engage in productive mathematical discussions (Kazemi & Hintz, 2014; Chapin et al., 2009). These teacher moves include: Individual Think Time, Turn and Talk, and the Four Rs (Repeat, Rephrase, Reword, and Record). Sense making is the focus of each move, rather than mere answer getting or answer telling. The moves are designed to provide the time, structure, and support to engage students in the co-construction of mathematical ideas and understanding (Kelemanik et al., 2016).

Facilitate Meaningful Discussions



Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students that counterexamples are a good way to show why you disagree with an idea or solution strategy.

Guide students to **Compare and Connect** the representations. Use turn and talk to help students think through their responses before sharing with the group.

ASK *How is the money raised each of the 8 days represented in each model? How is the goal represented? How is the money they still need to raise represented?*

LISTEN FOR Students should recognize that accurate models show 8 groups of 9 plus the quantity *m* equals 250. The quantity *m* represents the amount they still need to raise.

Picture It & Model It

If no student presented these models, have students analyze key features and then point out the ways each model represents:

- 8 groups of $9
- the amount they still need to raise, *m*
- the goal amount, $250

ASK *How many equal amounts of money did they raise? What is the equal amount? What is their goal? How much more do they need to raise?*

LISTEN FOR They raised 8 equal groups of $9. Their goal is $250 and they still need to raise *m* dollars to reach their goal.

For the bar model, prompt students to identify the way the information is organized.

- *What do the 9s at the beginning of the bar represent?*
- *How is the goal represented? What does the large section labeled m represent?*

For the notes and equation, prompt students to identify the reasoning process.

- *How do you know to multiply 8 and 9?*
- *Why does the equation show parentheses? Are they needed?*
- *How do you know to add in the second step?*

LESSON 18 DEVELOP

Explore different ways to understand how to solve two-step word problems using one equation.

Third graders at Spring Elementary School are raising money for the school library.
- **The goal is to raise $250.**
- **They raise $9 each day for 8 days in a row.**

How much more money, *m*, is needed to reach the goal? To solve this problem, write an equation for each step. Or write one equation that includes both steps.

PICTURE IT

You can use a diagram to show a two-step word problem.

250

9 9 9 9 9 9 9 9 m

MODEL IT

Use the diagram above to help write an equation for a two-step word problem.

The students raise $9 each day for 8 days. So, they have already raised 8×9 dollars.

They need a total of 250 dollars. They need to raise *m* more dollars.

The amount already raised plus *m* should equal 250.

Write this as one equation.

$(8 \times 9) + m = 250$

398

DIFFERENTIATION | EXTEND

SMP 5

Deepen Understanding

Words and Diagrams

As students learn to use appropriate tools, it is helpful for them to evaluate different tools for their usefulness in a given situation.

ASK *How is it a helpful strategy to use words and numbers to model the problem about raising money for the library? What can using a diagram show us that words and numbers may not?*

LISTEN FOR Words and numbers can help me to reason through the problem and record what I know and what is missing. I can work through each step as I think it through. The diagram shows me how the numbers connect and fit together. I can solve the problem any way I want. I can count by nines to 72 and then add on to get to 250, or I can find 8×9 and then subtract 72 from 250, or I can repeatedly subtract 9 eight times from 250.

Generalize *Why is it useful to know several ways to model the same problem?* You can pick the way that makes the most sense to you as you are solving. Also, using two different methods is a good way to check your work.

©Curriculum Associates, LLC Copying is not permitted.

Lesson 18 Solve Two-Step Word Problems Using the Four Operations 398

Downloaded by J. Rosenblum at Curriculum Associates. This resource expires on 6/30/2023.

Teacher moves like Individual Think Time and Turn and Talk help students develop their ideas and increase participation in discussions.

5. Pose Purposeful Questions

"Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships" (NCTM, 2014).

Effective mathematics teaching relies on questions that encourage students to explain and reflect on their own thinking and engage with and build upon their peers' thinking (NCTM, 2020a). It is through responding to posed questions that students can focus on key aspects of their meaning-making strategies. Purposeful questions allow teachers to formatively assess student understanding to determine what students know and where they need to adapt their lessons to meet their various learners' needs. These questions also help students make mathematical connections and support students in constructing their own questions.

Merely asking questions is not enough to ensure students make sense of mathematics and advance their reasoning. Both the types of questions teachers ask and the way in which they ask them should be considered (NCTM, 2020b). Posing questions in ways that are equitable means the teacher intentionally uses questioning to ensure each and every student progresses in their thinking, learning important and challenging mathematical ideas and developing a positive mathematical identity (Aguirre et al., 2013).

i-Ready Classroom Mathematics supports teachers in posing purposeful questions in numerous ways. During the Explore session, teachers pose questions to students to understand how they have made sense of new ideas while also digging deeper into the lesson content. Questions and prompts help students activate their prior knowledge and then build a bridge to the new content they will learn as the lesson progresses. During the Develop sessions, teachers pose questions to help students connect ideas. Questions help them connect their mathematical representations and strategies during classroom discussion, reflect on what they have learned, and enable them to apply their learning to new problems. Finally, in the Refine sessions, teachers use questions to help students continue to see and discover relationships in mathematics. Some specific supports for posing questions include:

- **Support Partner Discussion**: Teacher prompts to help students engage in meaningful peer discourse
- **Ask/Listen For**: These mathematical discourse questions, followed by expected student responses, support and facilitate whole class discussion. As students share their thinking, the questions can be used to make connections between student approaches and different models and representations, prompt justifications and critiques of approaches and solutions, and check conceptual understanding.
- **Monitor and Confirm Understanding**: These questions on the Connect It page ensure students have made sense of mathematical learning goals.
- **Facilitate Whole Class Discussion**: This series of related questions help illuminate the mathematical ideas of the lesson, prompting students to make connections and use that understanding to solve problems leading to abstract reasoning. These questions help students learn how to articulate a generalization of the mathematical concept.

Posing Purposeful Questions

Support Partner Discussion

Encourage students to use the term *tenths* as they discuss their solutions.

Support as needed with questions such as:

- *How did you get started?*
- *How would you describe your model?*

Common Misconception Look for students who write a fraction comparing the painted parts to the unpainted parts and write $\frac{7}{10}$ instead of comparing to the whole.

ASK *Where does your model show the total number of equal parts in the fence? the part Francisca paints? the part Nahele paints? the total number of tenths the two friends paint?*

LISTEN FOR Students should recognize that accurate responses include fractions with a denominator of 10 and representations that show equal parts. Responses may include 10 as the total number of equal parts, $\frac{3}{10}$ as the part Francisca paints, $\frac{4}{10}$ as the part Nahele paints, and $\frac{7}{10}$ as the total number of parts both friends paint.

i-Ready Classroom Mathematics *provides supports for language development and discourse including language routines, teacher moves, and conversation tips.*

6. Build Procedural Fluency from Conceptual Understanding

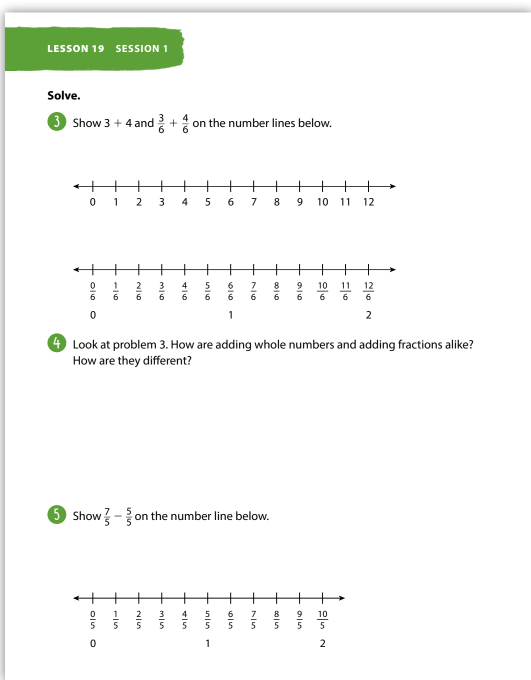
"Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems" (NCTM, 2014).

Effective mathematics teaching focuses on the development of both conceptual understanding and procedural fluency. Experts agree on the necessity of integrated and balanced development of both concepts and procedures in learning mathematics to support equitable learning (NCTM, 2023; National Mathematics Advisory Panel, 2008; National Research Council, 2001). They do recognize that procedural fluency follows and builds on a foundation of conceptual understanding, strategic reasoning, and problem solving.

When procedures are embedded into and grounded in concepts, students have more success with retaining information and can more effectively apply their knowledge and make sense of novel situations (NCTM, 2020b; Smith et al., 2017). When focusing prematurely on using procedures or memorizing facts, formulas, and algorithms without a solid conceptual understanding, students miss the opportunity to make meaning of their solutions and may have more difficulty approaching related but more complex problems. Building procedural fluency from conceptual understanding helps students better understand the purpose of mathematics, why it matters, and how to use mathematics as a tool to better understand the world around them.

In *i-Ready Classroom Mathematics*, students build their conceptual knowledge by reflecting on the concepts they already know and build upon them throughout the lesson. Ongoing practice helps students refine and reflect on concepts while solidifying procedures over time. Students have opportunities for daily practice to strengthen their conceptual understanding and build procedural fluency. For example, they might be asked to demonstrate different ways to show addition and then practice addition problems in their Student Worktext as they work to develop understanding. *i-Ready Classroom Mathematics* comes equipped with resources for teachers to facilitate Additional Practice, Fluency and Skills Practice, Cumulative Practice, digital Learning Games, Center Activities, and Interactive Practice with technology-enhanced items.

Deepen Conceptual Understanding



LESSON 19 SESSION 1

Solve.

3 Show $3 + 4$ and $\frac{3}{6} + \frac{4}{6}$ on the number lines below.

4 Look at problem 3. How are adding whole numbers and adding fractions alike? How are they different?

5 Show $\frac{7}{5} - \frac{5}{5}$ on the number line below.

Practice solidifies the connection between a new concept and prior knowledge.

7. Support Productive Struggle in Learning Mathematics

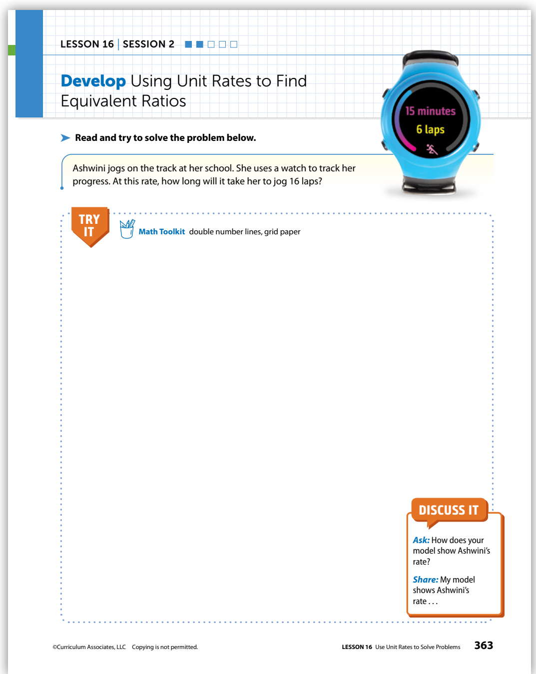
"Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships" (NCTM, 2014).

Effective practices in teaching mathematics provide students with regular opportunities to engage in productive struggle individually and with peers as they consider what they already know about mathematics and the new concepts they are learning. As students learn new ideas, they need time to work with them and develop conceptual understanding. Productive struggle is a necessary component of becoming an effective mathematician (Hiebert & Grouws, 2007). Teachers can best promote productive struggle by providing opportunities for reflection, questioning, recognizing, and acknowledging effort and providing sufficient time for students to engage. As students struggle productively, they learn to persevere and tackle challenges as they are confronted with them (Huinker & Bill, 2017). When teachers set high expectations for every student, they communicate that productive struggle is part of the process of learning mathematics and that students can succeed even if they encounter obstacles. This process builds confidence and agency.

To support productive struggle, *i-Ready Classroom Mathematics* embeds Try It opportunities throughout the program. Try It opportunities are so named to encourage students to jump in and grapple with the problem before being expected to be able to solve the problem. The Explore session provides an opportunity for students to use what they know to approach a new mathematical concept that they will be engaging with throughout the lesson. Students use a language routine to make sense of problems and then persevere in solving and supporting their thinking. As students think through the problem as a class and then try it on their own, first they activate prior knowledge and then hone their abilities to persist and persevere. After the class fully explores student-generated solutions, the explicit Picture It and Model It examples in the Student Worktext allow teachers to guide students' understanding and focus on the grade-level strategies of the lesson. This helps teachers validate, build on, and make the connection from student-generated solutions to the content of the lesson.

Within a given lesson in *i-Ready Classroom Mathematics*, each session (i.e., day) plays a different role in supporting student understanding. This provides students with a variety of experiences and gives them the time they need to think deeply about mathematics, ask questions, and reflect on what they are learning. For example, at the end of Connect It, students are asked to reflect on what they have learned, pose questions, and make real-world connections, further supporting productive struggle.

Building Perseverance



By having time to make sense of the problem as a class and then try it on their own first, students learn to tap into their existing knowledge and develop perseverance.

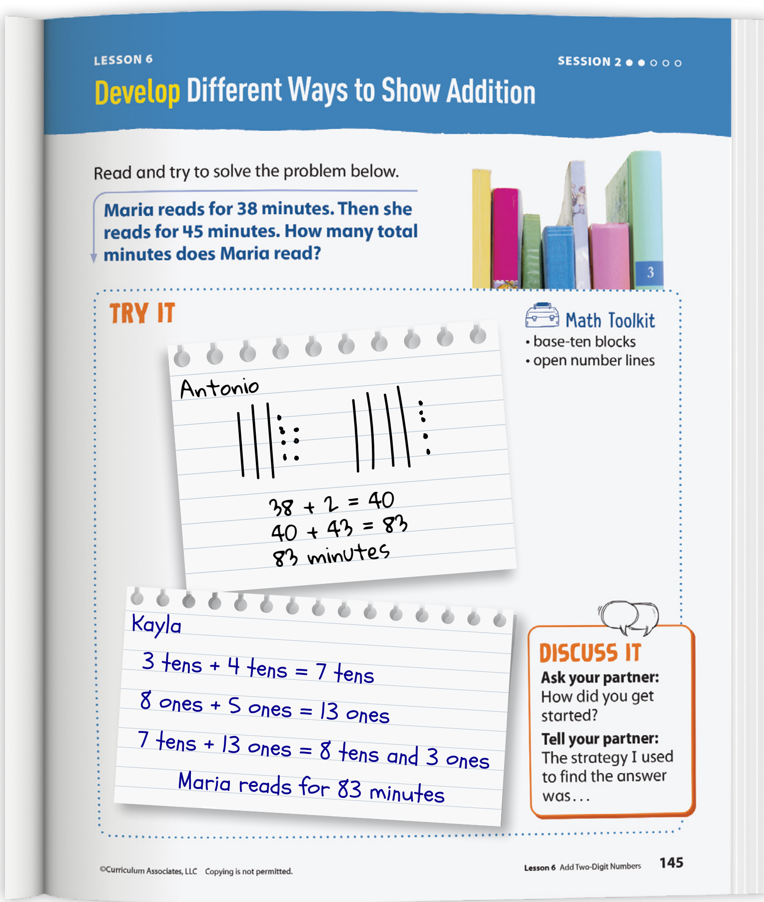
8. Elicit and Use Evidence of Student Thinking

"Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning" (NCTM, 2014).

Eliciting and using evidence of student thinking is an essential component of teaching and learning mathematics. It requires the focus to be placed on students' mathematical processing rather than simply focusing on their solutions. This process encourages teachers to consider their students' understanding of the task, patterns of reasoning, and how their ideas develop and transform over time (NCTM, 2014). Using evidence to assess students' understanding encourages teachers to look beyond the students' final answer and consider their process in how they construct their solutions. This provides critical information for how teachers can adjust, support, and differentiate ongoing instruction based on individual and group understanding and needs (NCTM, 2020b; Jacobs et al., 2010; Sleep & Boerst, 2010).

The *i-Ready Classroom Mathematics* program is designed to intentionally support teachers in eliciting examples of student thinking. From all three phases of the Try–Discuss–Connect framework and the embedded Ask/Listen For structure, to the partner and whole class discussions throughout lessons, students are regularly provided opportunities to share evidence of their thinking. This evidence supports teachers in making appropriate instructional decisions for each student.

Students Show Evidence of Their Thinking



In Try–Discuss–Connect, students can practice communicating their thinking in a variety of ways.

Cycles of Assessment

An evidence-based way to advance student learning is through using data to inform and guide teaching and learning (Slavin et al., 2013; Hamilton et al., 2009). It is important to note that what leads to significant gains in learning is not the data in and of itself, nor the intentional use of the data to formulate action plans. This must be accompanied by the implementation of research-informed instruction that supports the strengths and needs of the students who are identified in the data (Slavin et al., 2013).

With multiple ways to gather information on where students are before, during, and after teaching, *i-Ready Classroom Mathematics* assessment data provides valuable insights into student understanding. Building on the strengths of what students already know and what they are ready to learn, teachers can provide optimal levels of support for student growth. When viewed from an informative and asset-based lens, data can be used to provide targeted instructional recommendations that are teacher friendly, student friendly, comprehensible, and actionable in order to meet students where they are and prepare them for where they need to go next. *i-Ready Classroom Mathematics* supports the implementation of the instructional recommendations while providing ways to monitor student progress along the way. As such, educators have a roadmap for providing high-quality mathematics instruction to address unfinished learning effectively (Datnow et al., 2021).

In reconceptualizing assessment from a high-stakes approach to one that is more supportive of both student and teacher learning, NCTM positions assessment as "a process whose primary purpose is to gather data that support the teaching and learning of mathematics" (2014). This process is cyclical and flows from assessing to interpreting and taking action. In a call for assessments that honor students, families, and teachers, NCTM (2020a) proposes three key productive beliefs to hold about mathematics assessment:

- *The primary purpose of assessment in mathematics is to gather evidence of mathematical thinking, understanding, and reasoning to inform both instructional decisions and student and teacher learning.*
- *Multiple data sources at multiple points in time are needed to provide an accurate portrait of student and school mathematics performance to inform instructional and program improvements.*
- *Assessment is an ongoing process that should be an integral part of instruction, providing timely feedback to support students' learning and to inform instructional and program decisions.*

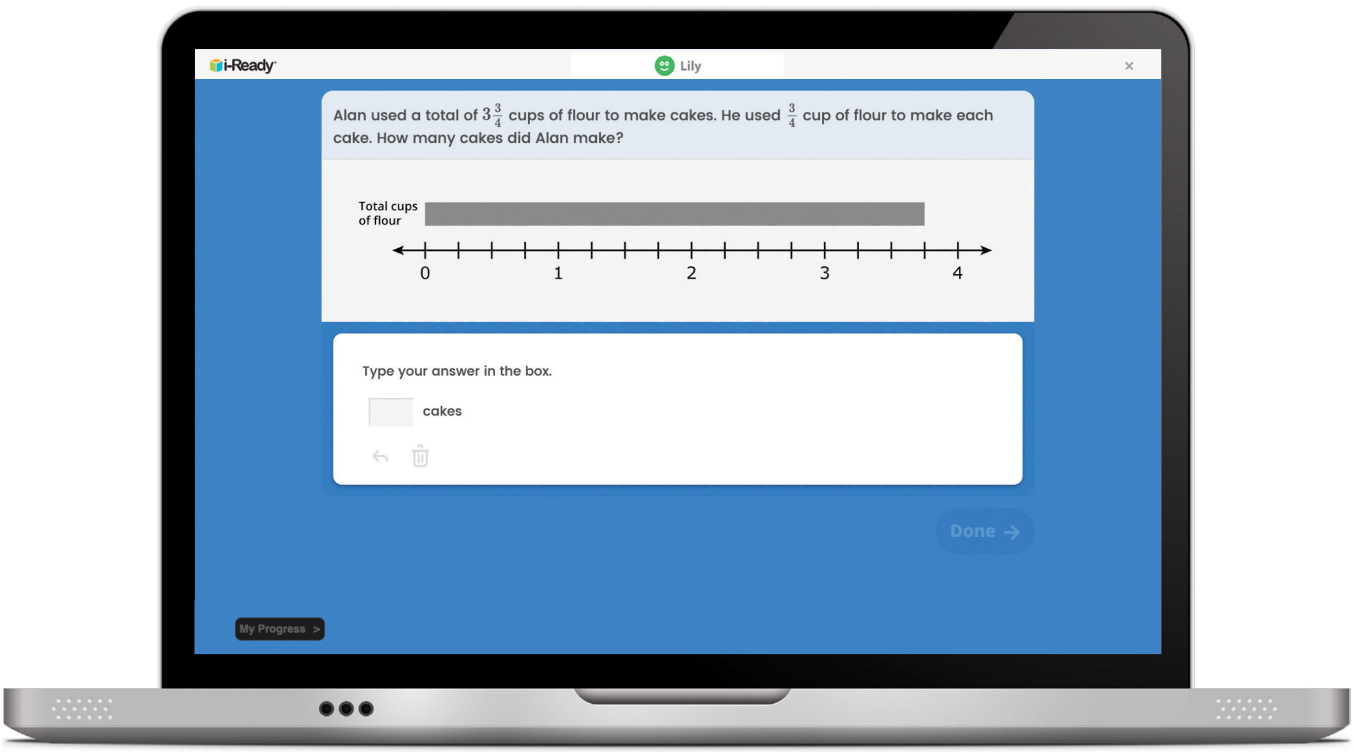
These productive beliefs about assessment are woven throughout *i-Ready Classroom Mathematics*, shaping the way it was designed and guiding the way it is to be implemented. As teachers use *i-Ready Classroom Mathematics* with their students, they will engage with assessment as an ongoing process in which data is continuously collected, analyzed, and utilized to inform instruction.

Using Data to Inform Instruction

A comprehensive understanding of student strengths and needs positions educators to more effectively tailor instruction to promote each student's success. Continually gathering evidence from multiple data sources to understand more fully where students are in their learning is a high-leverage practice for doing so (McLeskey et al., 2017). *i-Ready Classroom Mathematics* supports this process by providing multiple ways for educators to gather from students evidence they can use to inform their instruction.

Beginning with the *i-Ready Diagnostic*, this adaptive assessment pinpoints students' strengths and needs across grade-level mathematical skills and domains and provides educators with the information they need to plan and deliver instruction. It is criterion referenced so educators can compare students' performance against grade-level expectations, and it is norm referenced so educators can compare students' performance to other students. Educators can administer the Diagnostic at multiple points in the year to monitor student performance and growth across the mathematical skills and domains.

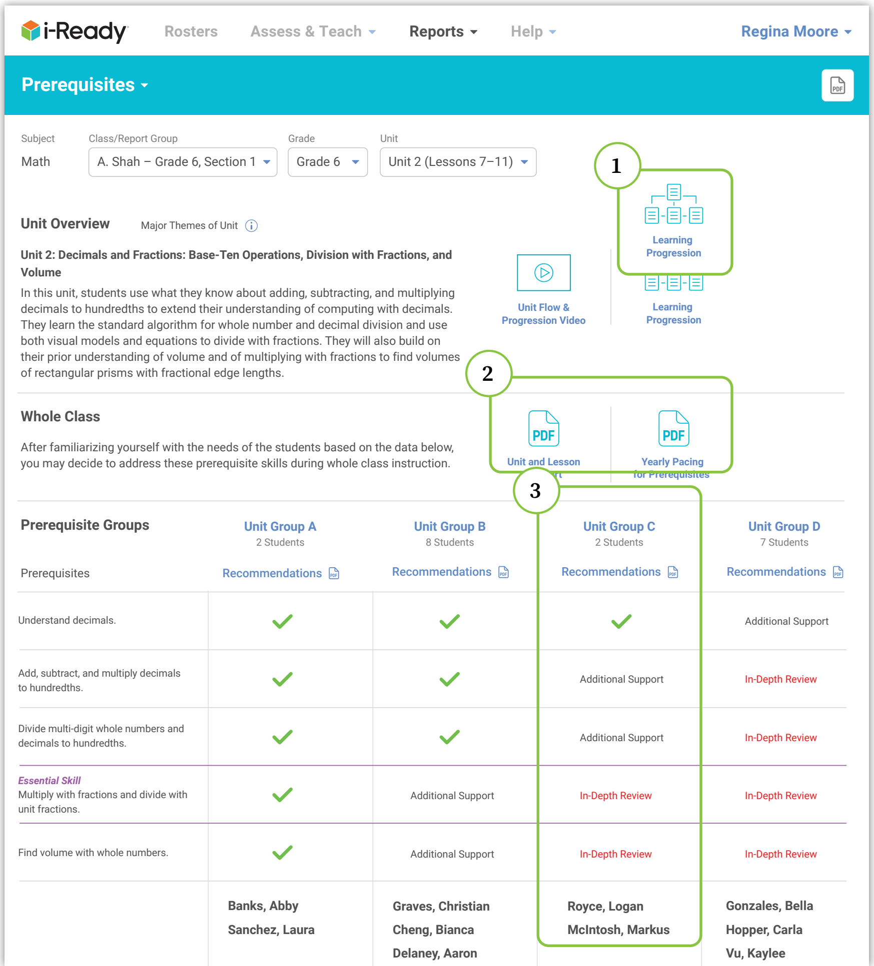
Identifying Students' Needs with the Diagnostic



The Diagnostic assessment is backed by extensive research, built and aligned to the high expectations of academic standards, and both criterion and norm referenced.

Based on results from the Diagnostic, educators can use the Prerequisites report to compare students' current level of performance with the prerequisite skills they need to have mastered to access the grade-level content in a lesson. In this way, educators can proactively address skills needed during instruction. The Learning Progression lays out the progression of standards going back two-plus years so educators know where to start and then where to go for each lesson. On-the-Spot Teaching Tips suggest additional scaffolding to support students with their learning as they engage in grade-level work, and the Yearly Pacing for Prerequisites provides guidance on when and how to use Prerequisite Lessons to support learning throughout the year.

Strategically Address Prerequisite Skills during Instruction



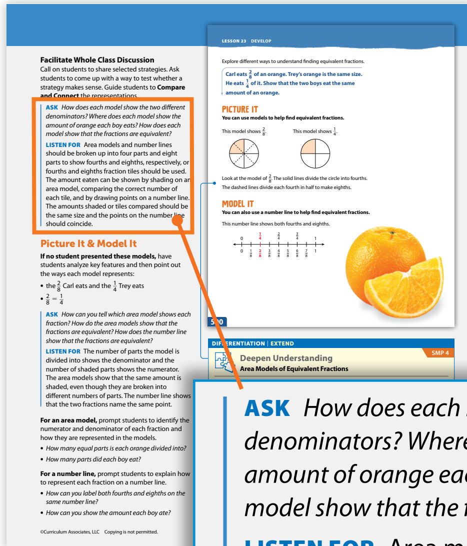
The Prerequisites report helps teachers to address unfinished learning with time-saving resources.

Whole Class Guidance and Pacing Support assists educators in integrating and scaffolding prerequisite skills into the grade-level content scope and sequence while teaching, and Small Group Resources assists in addressing specific, in-depth needs with targeted resources for teacher-led, partner, and independent activities. Opportunities are provided to monitor understanding and respond authentically to students in the moment. Just-in-time supports allow educators to reteach, reinforce, or extend learning using the activities outlined in the differentiation boxes in the Teacher's Guide, which can be found on Teacher Toolbox. Suggestions are given for where these activities can be used during instruction to support students' needs.

Informal Assessment Opportunities

Ongoing formative assessment allows educators to assess mathematical thinking, understanding, and reasoning and use this knowledge to plan instruction accordingly. *i-Ready Classroom Mathematics* supports teachers in becoming “students of their students” by helping them enhance their understanding of their students over time. In this way, teachers can better meet students where they are so they can grow. In a review that included 23 studies meeting rigorous standards of evidence, Mary Klute et al. (2017) found that teachers’ use of formative assessment had a significantly positive effect on student learning in mathematics. *i-Ready Classroom Mathematics* embeds formative assessment opportunities throughout the program. Specifically, students are asked to make connections between strategies and representations, reflect on what they have learned, and apply their learning to new problems and novel situations. Formative assessments within *i-Ready Classroom Mathematics* can be provided flexibly within a lesson.

Monitor Understanding



ASK *How does each model show the two different denominators? Where does each model show the amount of orange each boy eats? How does each model show that the fractions are equivalent?*

LISTEN FOR Area models and number lines should be broken up into four parts and eight parts to show fourths and eighths, respectively, or fourths and eighths fraction tiles should be used. The amount eaten can be shown by shading on an area model, comparing the correct number of each tile, and by drawing points on a number line. The amounts shaded or tiles compared should be the same size and the points on the number line should coincide.

Ask/Listen For questions provide opportunities for observation and discourse which can help teachers informally assess student understanding during the lesson.

Lesson Quizzes (i.e., paper) or Comprehension Checks (i.e., digital) can be given at multiple points—at the lesson and unit levels—to show students’ thinking and understanding around their learning. Both the paper and digital formats assess student progress toward mastery of session or unit content and are a way to identify where reteaching is needed. Exit Tickets and prompts at the close of a lesson are a quick formative assessment of each day’s learning and serve as an indicator of students’ progress toward mastery or partial mastery of the session’s learning goal.

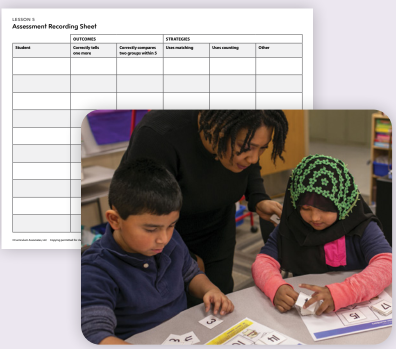
In addition, Digital Interactive Practice activities allow educators to assign online practice that gives students immediate, meaningful feedback. The Comprehension Check Results report shows student performance on these activities. Learning Games provide students with a fun way to practice mathematical concepts, while the corresponding Learning Games reports allow educators to see a real-time snapshot of performance on the standards and track students’ choices with respect to content that is challenging for them.

An Ongoing Process

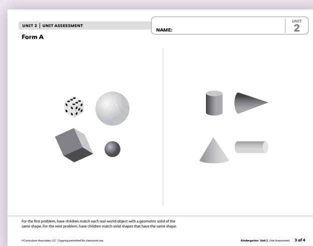
According to the Institute for Education Sciences handbook on using student data to inform instructional decision making, "Teachers should adopt a systematic process for using data in order to bring evidence to bear on their instructional decisions and improve their ability to meet students' learning needs" (Hamilton et al., 2009). The panel viewed this process as an ongoing cycle of instructional improvement that included three parts: collect and prepare a variety of qualitative and quantitative data about student learning; interpret data and develop hypotheses about how to improve student learning; and modify instruction to test hypotheses and increase student learning.

In *i-Ready Classroom Mathematics*, the multiple sources of data administered at multiple times allow educators to continually collect and interpret data about student learning, and it supports them in identifying what student strengths and needs should be targeted to improve learning. From here, the program provides multiple resources educators can choose from to differentiate their instruction so students have multiple ways to access and make sense of mathematical ideas as they grow and succeed in their learning.

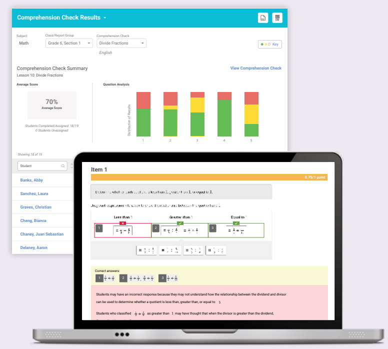
Assess Students' Understanding and Monitor Progress



Activity-Based Assessments (Grade K)



Paper/Pencil Assessments



Digital Assessments

Teachers choose how they want to evaluate students' strengths and dig deeper into their individual needs.

Committing to Learner Variability and Equity

In *i-Ready Classroom Mathematics*, students develop not only skills and conceptual understanding but also mindsets that support their journey to lifelong learning. With strategic opportunities that take into account the variability of all learners, instruction is flexible, equitable, and accessible to all. We strive to ensure that learners from all cultural identities, economic circumstances, and linguistic backgrounds, as well as those with disabilities, can engage with and see themselves reflected in our materials.

Differentiated Instruction

Differentiated instruction is an approach that focuses on students first and foremost. It acknowledges learner variability and strives to embed approaches into every lesson to ensure all students thrive. There are various moves a teacher can make in planning for and implementing lessons in a differentiated fashion, and *i-Ready Classroom Mathematics* supports teachers in doing this. Some moves include proactively modifying curricula to explore various teaching methods, resources, learning activities, or requirements for student deliverables in order to more effectively meet the learning goals of all students (Tomlinson et al., 2003).

The need for differentiated instruction is well established within the field of education. While the concept of differentiated instruction is widely accepted, teachers have reported that it can be difficult to implement in practice (Smale-Jacobse et al., 2019). Successful implementation of differentiated instruction is contingent on a number of factors. Teachers must have a clear understanding of their individual students and their learning needs. They develop this through formative assessment and ongoing progress monitoring. Students' needs are evolving and changing, which also necessitates frequent, ongoing assessment. High-quality, differentiated instruction is contingent on regular assessment of learning needs and flexible adaptations to meet those ever-changing needs, as opposed to an approach that relies on one-dimensional or fixed assessments of students that does not take into account student learning over time (Smets & Struyven, 2018).

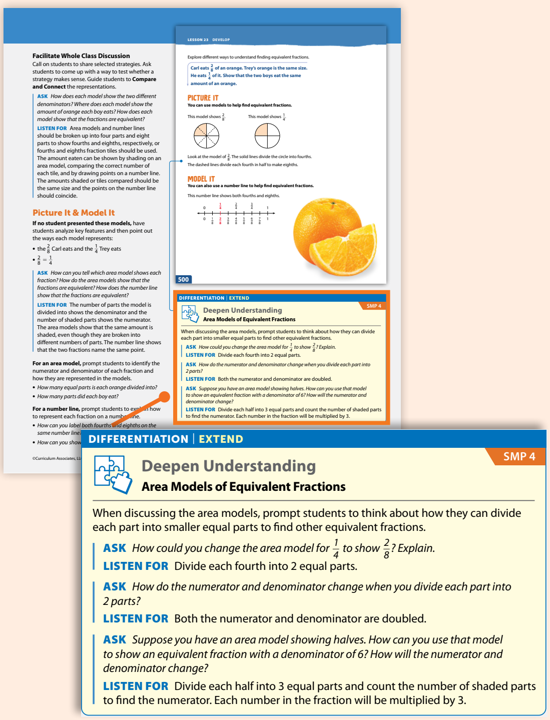
Resources for differentiation allow teachers to respond to insights from student data before, during, and after an *i-Ready Classroom Mathematics* lesson. Prior to the lesson, teachers have the opportunity to better understand the curriculum and learning goals so they can ensure each student has an opportunity to be successful. Based on their knowledge of students' readiness and other relevant characteristics (e.g., learning preference or interest), teachers may choose to differentiate the lesson content, process, learning environment, or learning time accordingly. *i-Ready Classroom Mathematics* offers opportunities for teachers to Reteach, Reinforce, and Extend in every session. During every session, options for differentiation are provided at specific hinge points in the lesson. After facilitating the lesson, teachers assess whether students have met the desired learning goals and determine instructional next steps based on needs for extension or reteaching.

Differentiation Options

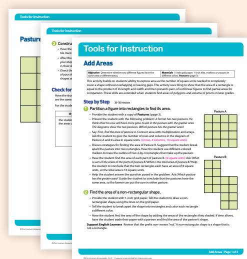
Differentiation Options during a Session

Additional Activities

Reteach, reinforce, or extend the learning using the activities provided in the yellow differentiation boxes in the Teacher's Guide.

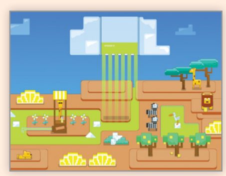


Differentiation Options after a Lesson



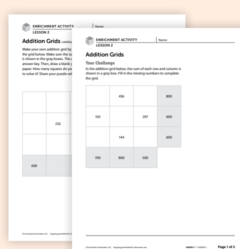
Reteach

Tools for Instruction are mini-lessons for reteaching lesson concepts.



Reinforce

Learning Games offer fun, challenging, and personalized practice and help students develop a growth mindset.



Extend

Enrichment Activities challenge students with higher-order thinking tasks.

Resources and support help teachers address students' needs and accelerate learning toward grade-level success.

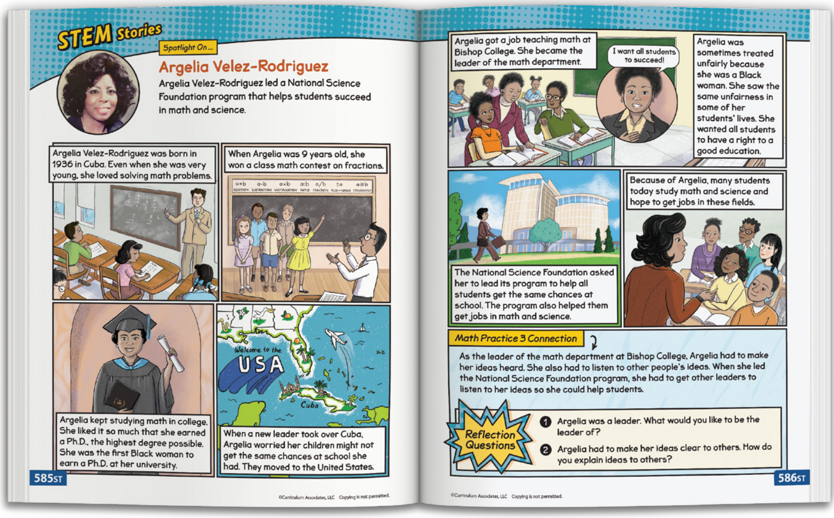
i-Ready Classroom Mathematics is built to support the teacher in successfully implementing differentiated instructional practices. Teachers monitor student work and their thinking from the beginning with assessments, Start activities in each lesson, and problem sets. Teachers seek to better grasp their students' understanding through observation of student work. Based on what they observe, teachers provide differentiated options for students to practice individually and in groups to support their individual development needs. The program also comes equipped with hands-on learning activities to reinforce and reteach concepts as well as additional mini-lessons for teachers to support students who need reteaching with explicit instruction. Additional practice problems are available for when students need more practice in specific areas. Finally, Challenge and Extend Activities deepen and extend students' understanding when they are ready to grapple with novel mathematical situations.

Universal Design for Learning

The UDL is a framework designed to improve and enhance teaching and learning based on what we know about how humans learn best. The UDL guidelines were created to "ensure that all learners can access and participate in meaningful, challenging learning opportunities" (CAST, 2020). When teachers apply the guidelines and concepts of the UDL into their lesson plans and practice, it can have a profound impact on all students' experiences and ongoing development (Spooner et al., 2007). *i-Ready Classroom Mathematics* provides a variety of ways for students to participate using what the UDL framework describes as "different modes of 'representation,' 'action and expression,' and 'engagement.'" It is important to recognize that the UDL framework reduces and removes barriers to allow all learners to access and engage with rigorous materials while maintaining high grade-level expectations (CAST, 2020). Built into *i-Ready Classroom Mathematics* are strategic opportunities informed by the UDL that further support learner variability.

- **Multiple Means of Representation** within the UDL framework refers to designing instructional materials that make content accessible to the greatest number of diverse learners by customizing the display of information, clarifying vocabulary and concepts, activating background knowledge, highlighting relationships across big ideas, and making content easily transferable for students (CAST, 2020).
    - **Comprehension Checks** have keyboard access, visual design enhancements (i.e., for minimum contrast and use of color), and audio supports.
    - **Student Bookshelf** has a wide range of supports such as annotation and text-to-speech features as well as various ways to navigate content.
    - **Interactive Practice** has embedded usability supports such as closed captioning, keyboard navigation, and visual design enhancements (i.e., for minimum contrast and use of color).
- **Multiple Means of Action and Expression** provides alternative communication methods (other than traditional written pencil-and-paper tests) for students to express, demonstrate, and monitor their learning (CAST, 2020; Courey et al., 2013). *i-Ready Classroom Mathematics* incorporates strategies for students to demonstrate their understanding in a variety of ways.
    - **Students engage in discourse to help them demonstrate their understanding.** For example, Individual Think Time gives them a chance to formulate thoughts to increase the quality and quantity of their expression of ideas. Turn and Talk is an opportunity for students to express their understanding and to learn from their peers. Both are valuable formative assessment tools for teachers as they seek to understand their students' way of thinking beyond traditional paper-and-pencil assessments.
    - **In Try It**, students make sense of problems in ways that make connections to who they are as individuals. This provides a real-world connection that helps make the concepts more relatable for students as well as allowing students to draw on their background knowledge.
- **Multiple Means of Engagement** not only invites student interest but also sparks motivation through creative, hands-on, meaningful instruction (CAST, 2020; Courey et al., 2013). *i-Ready Classroom Mathematics* uses many strategies to engage students to increase their autonomy with the taught skills.
    - **In the Explore session**, students build on their vocabulary by using a graphic organizer to review a previously learned term that plays a key role in the upcoming lesson. This helps students reflect on concepts they know and will build upon throughout the lesson.
    - **Throughout the program, students make connections to strategies, the underlying mathematics, and each other's thinking and ideas.** In the younger grades, they engage with stories and Connect to Culture activities that help them make personal connections and leverage their curiosity. In the older grades, they engage with STEM Stories and Real-World Connections that showcase the lives and contributions of role models in the fields of science, technology, engineering, and mathematics. These stories represent individuals with diverse backgrounds and provide real-life examples of mathematical practices in action. They serve to make learning more meaningful and applicable as students see themselves in their stories or learn about practical applications of mathematics in everyday society (Bishop, 1990).

Engagement Opportunities Motivate Students



STEM Stories provide engaging, relevant, and culturally diverse opportunities for students to connect with learning.

English Learners

More than five million students in the United States are designated as ELs (National Center for Education Statistics, 2022). Not only are ELs the fastest-growing population in the United States, they also represent a broad spectrum of learners with a wide range of backgrounds, experiences, languages, and academic proficiencies. ELs are a linguistically diverse and culturally rich population that bring many assets and competencies to the classroom. *i-Ready Classroom Mathematics* recognizes that all students are learners of academic language, which is an intrinsic part of mathematics. The program also considers ELs' unique learning needs to provide them with substantial opportunities to thrive. *i-Ready Classroom Mathematics* uses research-based best practices for teaching mathematics and supporting language development so ELs can access, participate in, and demonstrate their knowledge and skills in grade-level mathematics. Through high-quality mathematics instruction in *i-Ready Classroom Mathematics*, ELs acquire English-language skills, learn the language of mathematics, develop mathematical content knowledge, and apply learned concepts and skills to solving mathematics problems (Doabler et al., 2019). They engage in varied activities to enhance mathematics skills while applying and communicating their understanding in their developing language (Li et al., 2022; Moschkovich, 2002).

Using a variety of instructional methods, including peer-to-peer discourse, and guided practice has been shown to increase ELs' engagement in mathematics (Li et al., 2022). Reflecting the key principles for EL instruction summarized by Understanding Language at Stanford University Graduate School of Education (2017), *i-Ready Classroom Mathematics* integrates a variety of research-based language routines, teacher moves, and conversation tips to support ELs' access to grade-level mathematics and to participate fully within the lesson. The language routines help ensure students make sense of mathematics problems while also helping them express their ideas and participate more confidently in class discussions. During Try It, students also use and engage with visual representations of concepts. In the Model It and Picture It sections of the lesson, students experience both visual and abstract representations, including diagrams, graphic organizers, number lines, and equations. Presenting concepts in a variety of ways helps ELs create mental models that connect verbal information to a symbolic understanding of new material and play a critical role in solidifying ELs' mathematical understanding (Saxe & Sussman, 2019; Orosco, 2014). After the class fully explores student-generated solutions, the explicit Picture It and Model It examples in the Student Worktext gives teachers the option to guide students' understanding and focus on the grade-level strategies of the lesson. This helps teachers validate, build on, and make the connection from student-generated solutions to the content of the lesson.

Supports for Language Development

Try It	Discuss It	Connect It
Language Routines • Three Reads • Co-Craft Questions • Notice and Wonder • Say It Another Way **Teacher Moves** • Turn and Talk • Individual Think Time	**Language Routines** • Collect and Display • Compare and Connect **Teacher Moves** • Turn and Talk • Individual Think Time • Four Rs **Conversation Tips**	**Language Routines** • Collect and Display • Compare and Connect **Teacher Moves** • Turn and Talk • Individual Think Time • Four Rs

The language routines in Try–Discuss–Connect increase class participation and support students as they learn content, apply mathematical practices, and develop language.

The research-based mathematical language routines in *i-Ready Classroom Mathematics* are designed to support sense making, optimize output, cultivate conversation, and maximize lingustic and cognitive meta-awareness (Zwiers et al., 2017). These routines, such as Three Reads and Stronger and Clearer Each Time, are embedded within lesson activities and serve as scaffolded supports to facilitate discourse and the interaction needed to promote language development. In addition, teachers are guided to use a variety of teacher moves—Individual Think Time, Turn and Talk, and the Four Rs (Repeat, Rephrase, Reword, and Record)—to engage students in discourse and mathematical thinking. Using the routines and teacher moves throughout lesson activities provides opportunities for teachers to help students analyze, learn about, and practice the academic language that is integral to mathematics. Using these routines and teacher moves, students make sense of problems and justify their models and strategies, and make connections between them. As they engage in the routines and teacher moves, students practice applying new and familiar mathematical vocabulary and develop their ability to communicate ideas and concepts.

Each lesson provides targeted support for ELs along a continuum of proficiency levels in the form of the Differentiation: English Learners chart. The chart provides suggestions to help teachers scaffold the language in a specific problem or activity to ensure that all ELs can access and engage with the mathematics.

Differentiation for English Learners

DIFFERENTIATION | ENGLISH LEARNERS — Use with **Session 3** **Apply It**

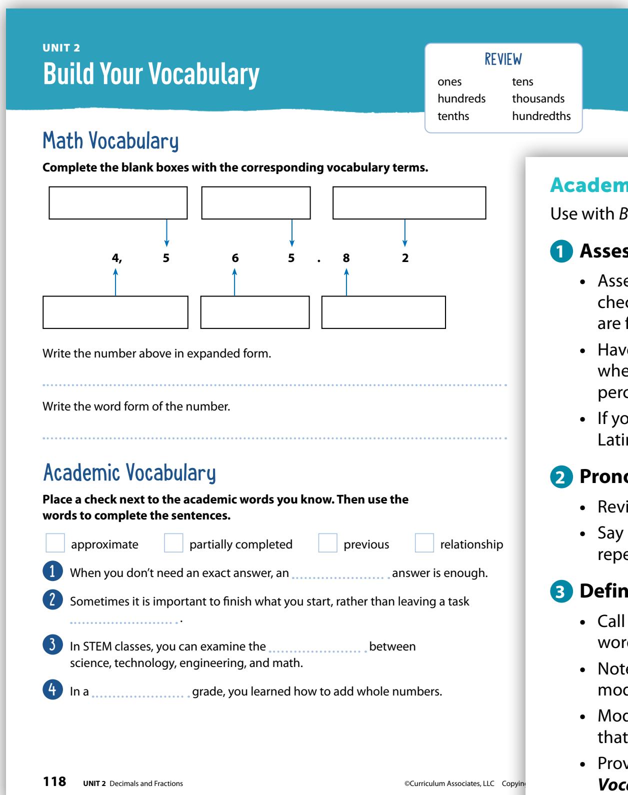
Levels 1–3: Speaking/Writing	**Levels 2–4:** Speaking/Writing	**Levels 3–5:** Speaking/Writing
Use **Stronger and Clearer Each Time** to refine ideas in Apply It problem 5. Have partners take turns orally explaining the directions as the other partner follows. Encourage speakers to use gestures, words, phrases, and modeling to communicate ideas. Next, direct student attention to part b of problem 4. Explain that directions often start with action words. Underline *draw* and *place*. Generate a list sentence starters that begin with verbs for students to use as they draft responses: • *Draw* ___. • *Label* ___. • *Line up* ___. • *Divide* ___.	Use **Stronger and Clearer Each Time** to refine students' ideas in Apply It problem 5. Have partners take turns giving directions to each other, adjusting language or adding details as needed. Point to problem 4 part b. Note that the sentences begin with action words/verbs, *draw* and *place*. Provide time for students to draft responses that begin with clear verbs. Choose a volunteer who wants to read their draft aloud and get feedback from the class. Prompt for more details by asking: *Where?* [on the top number line, on both lines] *Why?* [to show equivalent fractions, to line up the marks] *How?* [line up exactly, place directly below]	Use **Stronger and Clearer Each Time** to refine students' ideas in Apply It problem 5. Have students take turns giving oral directions to a partner. Place a divider or folder between them so the speaker cannot see the listener's drawing. Remind them to give specific directions so the listener can understand exactly what to do. Then have both students analyze the model and tell what parts of the description were clear or unclear. Facilitate a discussion about what makes the directions. Ideas may include: *strong verbs, precise language, key details* and *connecting words*. After students draft responses, have partners share and give feedback.

ELs are supported along a continuum of proficiency levels.

Peer interaction aims to increase students' flexibility with solving mathematics problems by having students work together to share ideas, connect concepts, and support their peers (Kersaint et al., 2013). Discuss It uses the language routines of Compare and Connect and Connect and Display as well as teacher moves such as the Four Rs (Repeat, Reword, Rephrase, and Record). When working with their peers, students process new information, gain experience with mathematical concepts, and practice talking through the process of solving problems, which has been proven to enhance students' mathematical learning (Mercer, 2008). Additionally, resources such as Discourse Cards and Multilingual Glossaries are available for teachers to use to help students verbalize and explore their mathematical ideas using content-specific language.

i-Ready Classroom Mathematics deepens conceptual understanding by building students' understanding and use of mathematical terms and academic language. While helpful for all learners, this is particularly important for ELs who may need support with core meanings of a word in their second language and may also have comprehension gaps (Barrow, 2014). To support ELs in learning how academic English works, the program provides support for developing vocabulary. At the beginning of each unit, students review mathematics terms from the prior unit in context, while they draw upon their prior knowledge to discuss and work with academic vocabulary. Session 1 provides a graphic organizer and activity that helps all students expand their conceptual understanding of a key mathematics term in the lesson.

Developing Mathematics and Academic Vocabulary


UNIT 2

Build Your Vocabulary

REVIEW

ones	tens
hundreds	thousands
tenths	hundredths

Math Vocabulary

Complete the blank boxes with the corresponding vocabulary terms.

4, 5 6 5 . 8 2

Write the number above in expanded form.

Write the word form of the number.

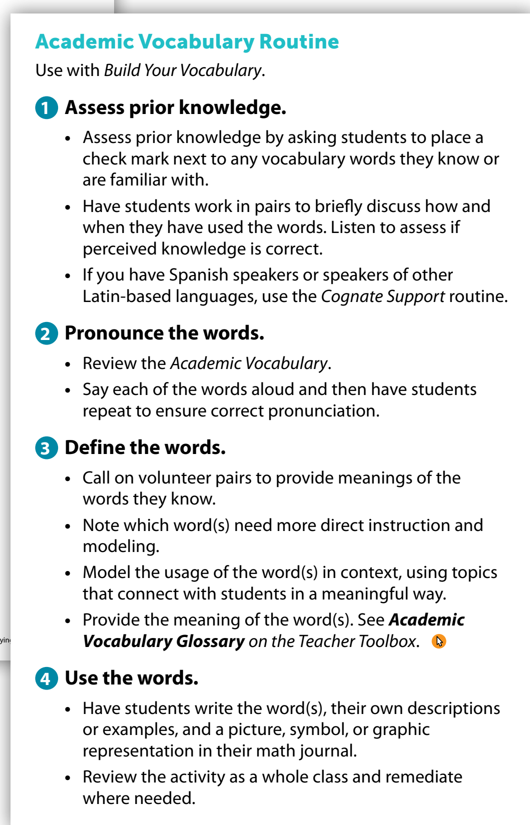
Academic Vocabulary

Place a check next to the academic words you know. Then use the words to complete the sentences.

approximate partially completed previous relationship

1. When you don't need an exact answer, an .... answer is enough.
2. Sometimes it is important to finish what you start, rather than leaving a task ....
3. In STEM classes, you can examine the .... between science, technology, engineering, and math.
4. In a .... grade, you learned how to add whole numbers.

118 UNIT 2 Decimals and Fractions ©Curriculum Associates, LLC Copyin


Academic Vocabulary Routine

Use with *Build Your Vocabulary*.

1. **Assess prior knowledge.**
   - Assess prior knowledge by asking students to place a check mark next to any vocabulary words they know or are familiar with.
   - Have students work in pairs to briefly discuss how and when they have used the words. Listen to assess if perceived knowledge is correct.
   - If you have Spanish speakers or speakers of other Latin-based languages, use the *Cognate Support* routine.
2. **Pronounce the words.**
   - Review the *Academic Vocabulary*.
   - Say each of the words aloud and then have students repeat to ensure correct pronunciation.
3. **Define the words.**
   - Call on volunteer pairs to provide meanings of the words they know.
   - Note which word(s) need more direct instruction and modeling.
   - Model the usage of the word(s) in context, using topics that connect with students in a meaningful way.
   - Provide the meaning of the word(s). See ***Academic Vocabulary Glossary*** *on the Teacher Toolbox.*
4. **Use the words.**
   - Have students write the word(s), their own descriptions or examples, and a picture, symbol, or graphic representation in their math journal.
   - Review the activity as a whole class and remediate where needed.

Students are encouraged to engage in rigorous mathematics and to communicate effectively.

Additionally, the Develop Academic Language feature in every Develop session provides guidance for learning about language at the word, sentence, and discourse levels so all students can engage in rigorous mathematics. For example, at the word level, the Develop Academic Language feature explores multiple-meaning words, word families, and word parts. At the sentence level, students learn from how to interpret and compose complex sentences in the Develop Academic Language feature, and Discourse Cards provide sentence starters and questions to sustain conversation. The Develop Academic Language feature also provides guidance on discourse skills to help students carry out the mathematical practices. Students learn how to listen actively, explain and justify their ideas, agree and disagree, and make connections between ideas, strategies, and models.

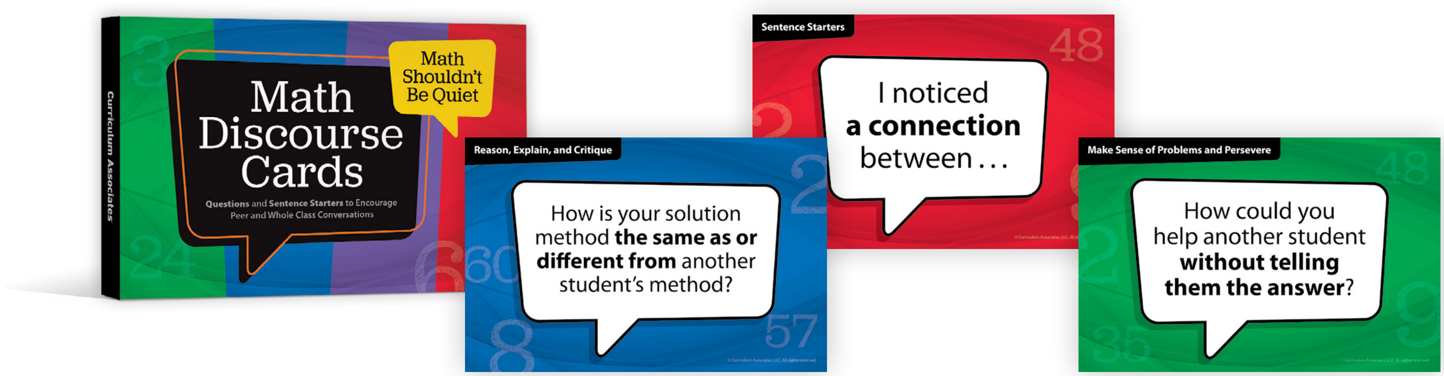
Language and Discourse Support

DEVELOP ACADEMIC LANGUAGE

WHY? Clarify understanding of the multiple-meaning word *left*.

HOW? Facilitate a discussion about the definitions of the word *left*. Affirm ideas that *left* can mean a direction, the past tense of the verb leave, or that something remains unused. Call on volunteers to share examples using each meaning. Explain that readers can use context clues to understand what the word means in the situation, as in the Try It problem.

Activities and support at the word/phrase, sentence, and discourse levels help all students engage in rigorous mathematics and communicate effectively.



Responsive Teaching and Learning

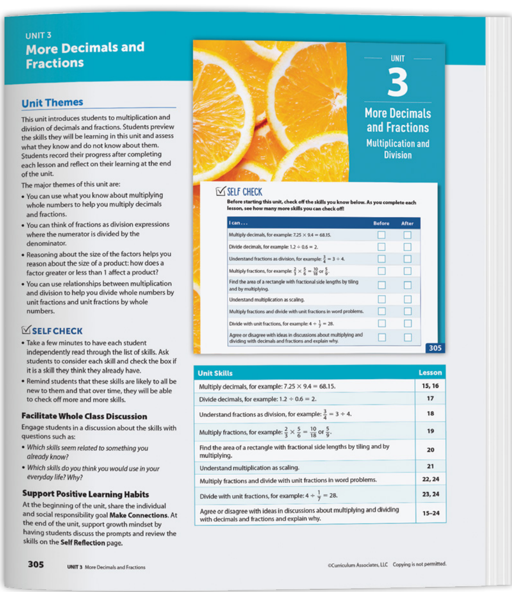
i-Ready Classroom Mathematics embeds best practices of culturally and linguistically responsive learning to support every learner in developing a sense of ownership of and agency in mathematics (Ellis, 2021; Aguirre & Zavala, 2013). Validating and affirming students' home cultures can meaningfully develop their existing knowledge and connect it to academic content within the school context (Hollie, 2015; Muhammad, 2020).

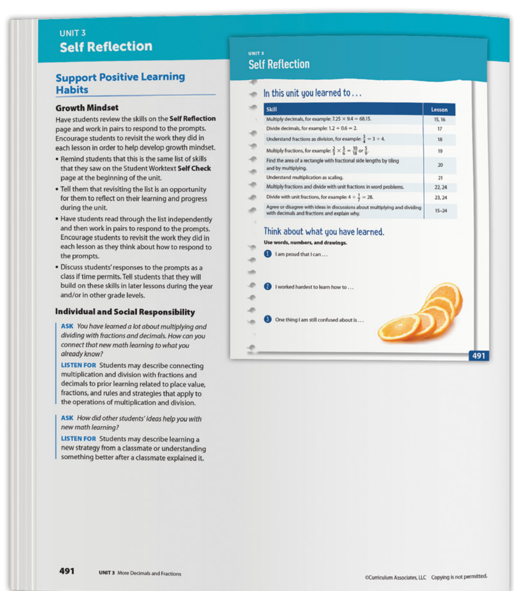
i-Ready Classroom Mathematics helps validate and affirm students' identities by including diverse problem contexts and providing teachers with the Connect to Culture and Protocols for Engagement support. It does so by honoring students' cultural and linguistic knowledge as strengths on which teachers can build their instruction (Au, 2006; Gay, 2018; Kolovou, 2022; Ladson-Billings, 2009; Nieto, 2009).

Individual and Social Responsibility

i-Ready Classroom Mathematics includes opportunities for students to promote positive learning habits and growth mindsets by reflecting on their understanding and developing self-awareness, self-management, social awareness, communication skills, and responsible decision making. Students regularly have opportunities to check their work and to consider what they already know before starting a lesson as well as to return to the key concept at the end of the unit for them to reflect on their progress. Further opportunities for self-reflection prompt students to consider what they have learned over multiple lessons. These reflections are embedded throughout the program and work to further build confidence and a strong sense of accomplishment. Through this process, students learn valuable skills for developing their conceptual understanding of mathematics and helping them continue to grow and take risks as confident and capable mathematicians (Abdulrahim & Orosco, 2019).

Support Student Agency





Students reflect on what they already know at the beginning of a unit and on their progress at the end of a unit.

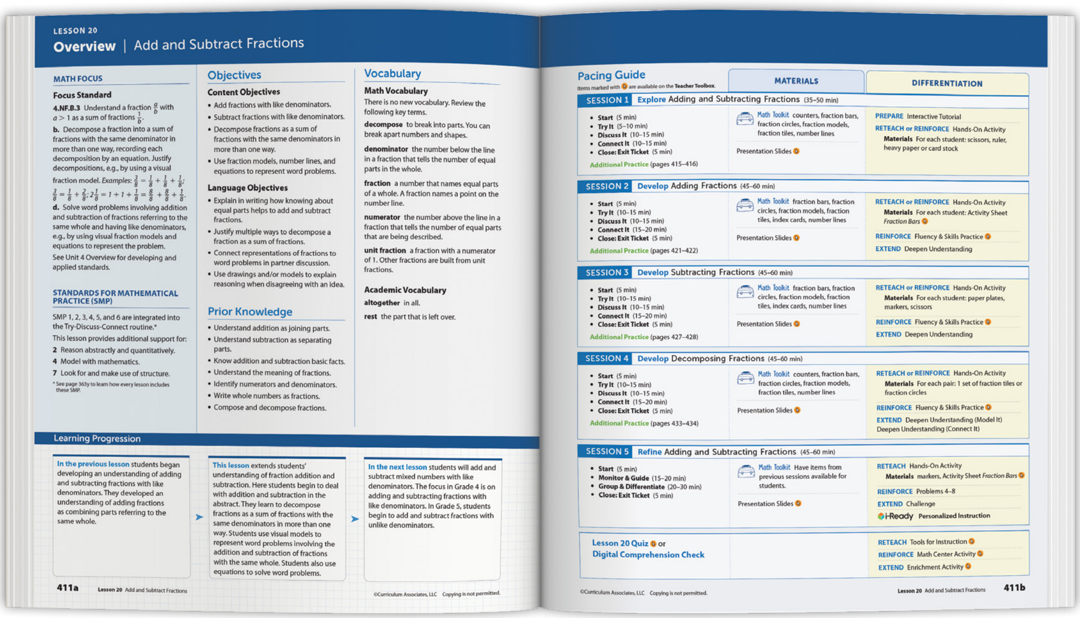
Supporting Teachers

With intentional flexibility and a wealth of time-saving resources that are strategically organized for ease of use and convenience, teachers can find everything they need in *i-Ready Classroom Mathematics* to lead their students to success—all in one place. Moreover, the resources and supports are designed such that each teacher's unique needs can be met. Student outcomes are improved when teachers are supported in a way that can flexibly adapt to their strengths and needs as well as to those of their students (Toropova et al., 2021).

Instructional Planning

Educators benefit from support in the implementation of teaching practices in ways that are responsive to the unique needs of their students and that skillfully and knowledgeably support student learning (Grossman, 2018). To assist with planning instruction efficiently and effectively, the Lesson Overview pages in *i-Ready Classroom Mathematics* cover what will be needed to carry out each lesson. The Teacher's Guide provides embedded support with strategies, prompts, and in-the-moment guidance for each lesson. The Teacher Toolbox provides a digital collection of instructional resources for teachers to access grade-level materials for further support.

Plan Lessons with Ease

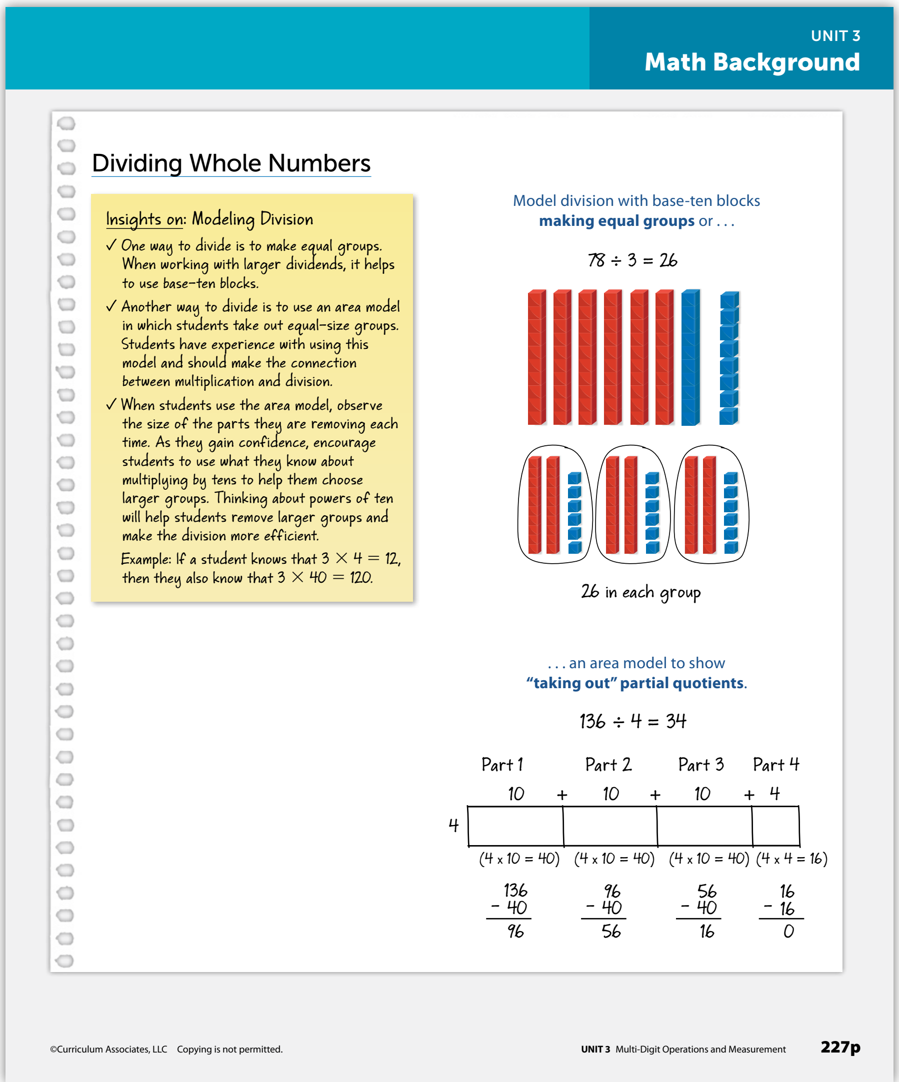


The Lesson Overview supports teachers in quickly understanding the lesson, so that they can effectively plan instruction.

Professional Learning

Professional learning is an ongoing process that continues throughout an educator's career and plays a role in their satisfaction and success (Ping et al., 2018). The professional learning opportunities in *i-Ready Classroom Mathematics* are designed to enhance the art and science of teaching mathematics and thus empower teachers to continue to grow and develop in their craft. The Teacher's Guide includes examples of how the models and strategies used in a particular unit fit into the learning progression, and support videos help teachers move through the progression in a way that keeps them on track to deliver all grade-level content by the end of the year. With Success Central, teachers have on-demand access to everything they need to know to implement the program successfully. There are also opportunities for ongoing, classroom-focused professional development, in which experienced educators deliver immersive experiences that support teachers in using students' thinking and mathematical practices to transform their mathematics classrooms.

Professional Learning Opportunities


UNIT 3
Math Background

Dividing Whole Numbers

Insights on: Modeling Division

✓ One way to divide is to make equal groups. When working with larger dividends, it helps to use base-ten blocks.

✓ Another way to divide is to use an area model in which students take out equal-size groups. Students have experience with using this model and should make the connection between multiplication and division.

✓ When students use the area model, observe the size of the parts they are removing each time. As they gain confidence, encourage students to use what they know about multiplying by tens to help them choose larger groups. Thinking about powers of ten will help students remove larger groups and make the division more efficient.

Example: If a student knows that $3 \times 4 = 12$, then they also know that $3 \times 40 = 120$.

Model division with base-ten blocks **making equal groups** or . . .

$78 \div 3 = 26$

26 in each group

. . . an area model to show **"taking out" partial quotients**.

$136 \div 4 = 34$

	Part 1	Part 2	Part 3	Part 4
	10 +	10 +	10 +	4
4				
	(4 x 10 = 40)	(4 x 10 = 40)	(4 x 10 = 40)	(4 x 4 = 16)
	136 − 40 = 96	96 − 40 = 56	56 − 40 = 16	16 − 16 = 0

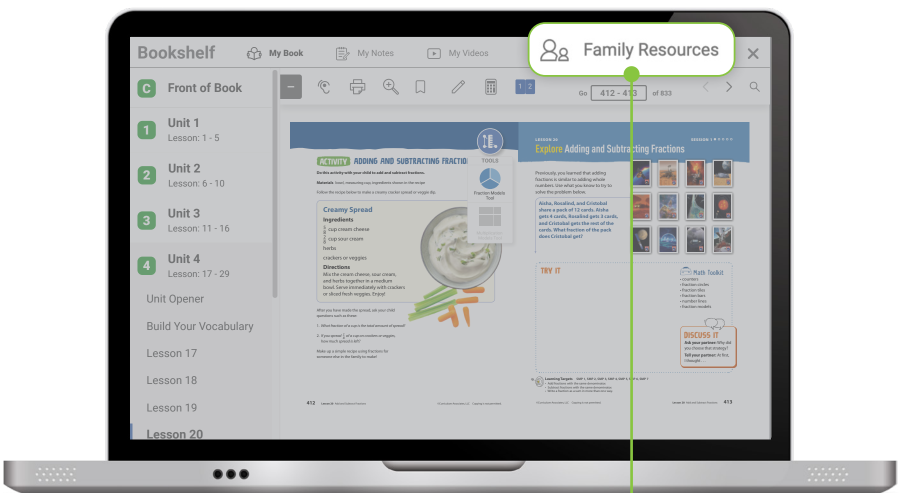
©Curriculum Associates, LLC Copying is not permitted. UNIT 3 Multi-Digit Operations and Measurement 227p

The Math Background in the Teacher's Guide shows how the models and strategies used in the unit fit into the learning progression.

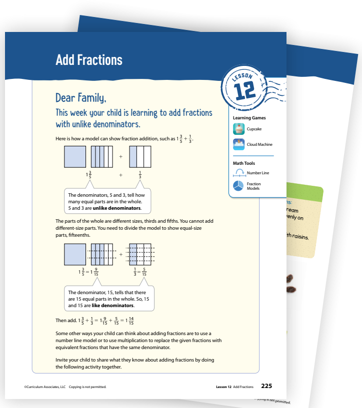
Family Engagement

i-Ready Classroom Mathematics strives to bring classrooms and communities together by providing opportunities to extend learning beyond the classroom. Family engagement is a cornerstone of good school organization, as a goal-linked family and community engagement are key to student success in school (Epstein, 2018). *i-Ready Classroom Mathematics* supports this partnership with an abundance of resources that families can use at home to support their students' mathematical growth. For example, the Student Bookshelf provides resources to understand mathematical ideas, such as a digital version of the Student Worktext, which can inform discourse at home. Family Letters, available in up to 11 languages for every lesson, provide mathematics background and an activity related to each lesson. Unit Flow & Progression Videos, which have closed captioning available in English and Spanish, help families support their students with the ideas and concepts taught in the curriculum. The Family Center website, available in English and Spanish, is dedicated to helping families explore the program and support their students at home. With Success Central, teachers can access resources to make family communication easier. Some resources include an Introduction Letter that introduces the curriculum and a Family Night presentation that gives an overview of the program.

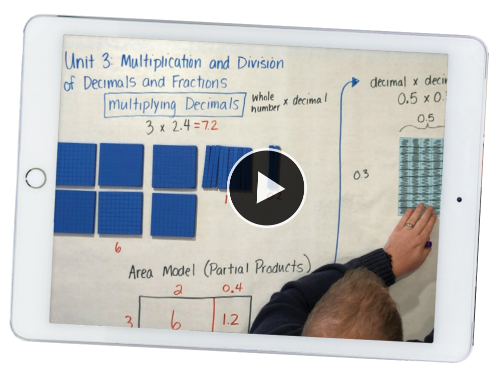
Resources to Engage Families



The program includes resources to help teachers make family communication easier, including Family Letters and a Family Night presentation, as well as to help families understand mathematics ideas through digital access to the Student Worktext and Unit Flow & Progression Videos in English and Spanish.



Family Letters, available in 11 languages for every lesson, provide mathematics background and an activity related to the lesson.



Unit Flow & Progression Videos help families support their student with the ideas and concepts taught in the curriculum. Closed captioning is available in English and Spanish.

Conclusion

A comprehensive core mathematics program, *i-Ready Classroom Mathematics* is built on research-based best practices to provide a balanced approach to mathematics instruction that focuses on building conceptual understanding of mathematical concepts, developing procedural fluency around them, and demonstrating learning through applications to new situations. Centered around the principles that students are at the heart of learning, high-impact strategies are a must, instructional instructions should be informed by data, and teachers thrive with support and resources to meet their students' needs, *i-Ready Classroom Mathematics* empowers students with the conceptual understanding and procedural fluency necessary to grow in mathematics and take ownership of their learning.

References

Abdulrahim, N., & Orosco, M. J. (2019). Culturally responsive mathematics teaching: A research synthesis. *The Urban Review, 52*(1), 1–25.

Aguirre, J., Mayfield-Ingram, K., & Martin, D. (2013). *The impact of identity in K–8 mathematics: Rethinking equity-based practices*. National Council of Teachers of Mathematics.

Aguirre, J. M., & Zavala, M. (2013). Making culturally responsive mathematics teaching explicit: A lesson analysis tool. *Pedagogies: An International Journal, 8*(2),163–190.

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52*(3), 215–241.

Au, K. H. (2006). *Multicultural issues and literacy achievement.* Lawrence Erlbaum Associates.

Barrow, M. A. (2014). Even math requires learning academic language. *Phi Delta Kappan, 95*(6), 35–38.

Bishop, R. S. (1990). Mirrors, windows, and sliding glass doors. *Perspectives: Choosing and Using Books for the Classroom, 6*(3).

Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. *Teachers College Record, 110*(3), 608–645.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic & algebra in elementary school.* Heinemann.

CAST. (2020). *UDL guidelines*. CAST.

Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn: 2nd ed.* Math Solutions.

Courey, S. J., Tappe, P., Siker, J., & LePage, P. (2013). Improved lesson planning with Universal Design for Learning (UDL). *Journal of the Teacher Education Division of the Council for Exceptional Children, 36*(1), 7–27.

Datnow, A., Lockton, M., & Weddle, H. (2021). Capacity building to bridge data use and instructional improvement through evidence on student thinking. *Studies in Educational Evaluation, 69*, 100869.

Doabler, C. T., Clarke, B., Kosty, D., Smolkowski, K., Kurtz-Nelson, E., Fien, H., & Baker, S. K. (2019). Building number sense among English Learners: A multisite randomized controlled trial of a Tier 2 kindergarten mathematics intervention. *Early Childhood Research Quarterly, 47*, 432–444.

Ellis, M. (2021). *Knowing and valuing every learner: Nora's story about culturally responsive mathematics teaching.* Curriculum Associates.

Epstein, J. L. (2018). School, family, and community partnerships in teachers' professional work. *Journal of Education for Teaching, 44*(3), 397–406.

Fuson, K. C., Kalchman, M., & Bransford, J. D. (2005). Mathematical understanding: An introduction. In M. S. Donovan, & J. Bransford (Eds.), *How students learn: History, mathematics, and science in the classroom* (pp. 217–256). National Academies Press.

Fuson, K. C., & Murata, A. (2007). Integrating the NRC principles and the NCTM process standards: Cognitively guided teaching to individualize instruction within whole-class activities and move all students within their learning path. *National Council of Supervisors of Mathematics Journal, 10*(1), 72–91.

Gay, G. (2018). *Culturally responsive teaching: Theory, research, and practice*. Teachers College Press.

Grossman, P. (Ed.). (2018). *Teaching core practices in teacher education*. Harvard Education Press.

Hamilton, L., Halverson, R., Jackson, S. S., Mandinach, E., Supovitz, J. A., & Wayman, J. C. (2009). *Using student achievement data to support instructional decision making (NCEE 2009-4067)*. US Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance.

Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement.* Routledge.

Haystead, M. W., & Marzano, R. J. (2009). *Meta-analytic synthesis of studies conducted at Marzano Research Laboratory on Instructional Strategies*. Marzano Research Laboratory.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Heinemann.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 371–404). Information Age.

Hiebert, J., Morris, A. K., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of Teacher Education, 58*(1), 47–61.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal, 30*(2), 393–425.

Hollie, S. (2015). *Strategies for culturally and linguistically responsive teaching and learning.* Shell Education.

Huinker, D., & Bill, V. (2017). *Taking action: Implementing effective mathematics teaching practices: K–Grade 5*. NCTM.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169–202.

Kazemi, E., & Hintz, A. (2014). *Intentional talk: How to structure and lead productive mathematical discussions*. Stenhouse Publishers.

Kelemanik, G., Lucenta, A., & Janssen Creighton, S. (2016). *Routines for Reasoning.* Heinemann.

Kersaint, G., Thompson, D. R., & Petkova, M. (2013). *Teaching mathematics to English Language Learners: 2nd ed.* Routledge.

Klute, M., Apthorp, H., Harlacher, J., & Reale, M. (2017). *Formative assessment and elementary school student academic achievement: A review of the evidence*. US Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Central.

Kolovou, M. (2022). Embracing culturally relevant education in mathematics and science: A literature review. *The Urban Review*, *55*(1), 133–172.

Ladson-Billings, G. (2009). *The dreamkeepers: Successful teachers of African American children*. Jossey-Bass.

Lesh, R., Post, T. R., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janiver (Ed.), *Problems of Representations in the Teaching and Learning of Mathematics* (pp. 33–40). Erlbaum.

Li, J-T., Arizmendi, G. D., & Swanson, H. L. (2022). The influence of teachers' math instructional practices on English Learners' reading comprehension and math problem-solving performance in Spanish and English. *International Journal of Bilingual Education and Bilingualism, 25*(10), 3614–3630.

Marzano, R. J. (2003). *What works in schools: Translating research into action*. ASCD.

McLeskey, J., Barringer, M-D., Billingsley, B., Brownell, M., Jackson, D., Kennedy, M., Lewis, T., Maheady, L., Rodriguez, J., Scheeler, M. C., Winn, J., & Ziegler, D. (2017). *High-leverage practices in special education*. Council for Exceptional Children, CEEDAR Center.

McTighe, J., & Wiggins, G. (2013). *Essential questions: Opening doors to student understanding.* ASCD.

Mercer, N. (2008). Talk and the development of reasoning and understanding. *Human Development, 51*(1), 90–100.

Michaels, S., O'Connor, C., & Resnick, L. B. (2008). Deliberative discourse idealized and realized: Accountable talk in the classroom and in civic life. *Studies in Philosophy and Education, 27*(4), 283–297.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4*(2/3), 189–212.

Muhammad, G. (2020). *Cultivating genius: An equity framework for culturally and historically responsive literacy.* Scholastic.

National Center for Education Statistics. (2022). *Digest of education statistics.* Author.

National Mathematics Advisory Panel. (2008). *The Final Report of the National Mathematics Advisory Panel*. US Department of Education.

National Research Council. (2001). *Knowing and learning mathematics for teaching: Proceedings of a workshop*. National Academies Press.

NCTM. (2014). *Principles to actions: Ensuring mathematical success for all*. Author.

NCTM. (2020a). *Catalyzing change in early childhood and elementary mathematics*. Author.

NCTM. (2020b). *Catalyzing change in middle school mathematics*. Author.

NCTM. (2023). *Procedural fluency: Reasoning and decision-making, not rote application of procedures position.* National Council of Teachers of Mathematics.

Nieto, S. (2009). *The light in their eyes: Creating multicultural learning communities*. Teachers College Press.

Orosco, M. J. (2014). A math intervention for third grade Latino English Language Learners at risk for math disabilities. *Exceptionality, 22*(4), 205–225.

Pape, S. J., & Tchoshanov, M. A. (2001). The role of representations in developing mathematical understanding. *Theory into Practice, 40*(2), 118–127.

Ping, C., Schellings, G., & Beijaard, D. (2018). Teacher educators' professional learning: A literature review. *Teaching and Teacher Education, 75*, 93–104.

Saxe, G. B., & Sussman, J. (2019). Mathematics learning in language inclusive classrooms: Supporting the achievement of English Learners and their English proficient peers. *Educational Researcher, 48*(7), 452–465.

Slavin, R. E., Cheung, A., Holmes, G., Madden, N. A., & Chamberlain, A. (2013). Effects of a data-driven district reform model on state assessment outcomes. *American Educational Research Journal, 50*(2), 371–396.

Sleep, L., & Boerst, T. A. (2010). Preparing beginning teachers to elicit and interpret students' mathematical thinking. *Teaching and Teacher Education, 28*(7), 1038–1048.

Smale-Jacobse, A. E., Meijer, A., Helms-Lorenz, M., & Maulana, R. (2019). Differentiated instruction in secondary education: A systematic review of research evidence. *Frontiers in Psychology, 10*.

Small, M. (2017). *Good questions: Great ways to differentiate mathematics instruction in the standards-based classroom: 3rd ed*. Teachers College Press.

Smets, W., & Struyven, K. (2018). Realist review of literature on catering for different instructional needs with preteaching and extended instruction. *Education Sciences, 8*(3), 113.

Smith, M., Steele, M., & Raith, M. L. (2017). *Taking action: Implementing effective mathematics teaching practices*. NCTM.

Spooner, F., Baker, J., Harris, A. A., Ahlgrim-Delzell, L., & Browder, D. M. (2007). Effects of training in Universal Design for Learning on lesson plan development. *Remedial and Special Education, 28*(2), 108–116.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2*(1), 50–80.

Stein, M. K., Smith, M. S., Henningsen, M., & Silver, E. A. (2009). *Implementing standards-based mathematics instruction: A casebook for professional development: 2nd ed*. Teachers College Press.

Stylianou, D. A, & Silver, E. A. (2004). The role of visual representations in advanced mathematical problem solving: An examination of expert–novice similarities and differences. *Mathematical Thinking and Learning, 6*(4), 353–387.

Tomlinson, C. A., Brighton, C., Hertberg, H., Callahan, C. M., Moon, T. R., Brimijoin, K., Conover, L. A., & Reynolds, T. (2003). Differentiating instruction in response to student readiness, interest, and learning profile in academically diverse classrooms: A review of literature. *Journal for the Education of the Gifted, 27*(2/3), 119–145.

Toropova, A., Myrberg, E., & Johansson, S. (2021). Teacher job satisfaction: The importance of school working conditions and teacher characteristics. *Educational Review, 73*(1), 71–97.

Turner, E., Dominguez, H., Maldonado, L., & Empson, S. (2013). English Learners' participation in mathematical discussion: Shifting positioning and dynamic identities. *Journal for Research in Mathematics Education, 44*(1), 199–234.

Webb, D. C., Boswinkel, N., & Dekker, T. (2008). Beneath the tip of the iceberg: Using representations to support student understanding. *Mathematics Teaching in the Middle School, 14*(2), 110–113.

Wiliam, D. (2011). *Embedded formative assessment*. Solution Tree.

Wood, M. B., Sheldon, J., Felton-Koestler, M. D., Oslund, J., Parks, A. N., Crespo, S., & Featherstone, H. (2019). 8 teaching moves supporting equitable participation. *Teaching Children Mathematics, 25*(4), 218–223.

Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). *Principles for the design of mathematics curricula: Promoting language and content development.* Understanding Language, Stanford Center for Assessment, Learning, and Equity. Stanford University Graduate School of Education.

i-Ready Classroom Mathematics

Built to address the rigor of the new standards, *i-Ready* helps students make real gains. *i-Ready* collects a broad spectrum of rich data on student abilities that identifies areas where a student needs support, measures growth across a student's career, supports teacher-led differentiated instruction, and provides a personalized instructional path within a single online solution.

To learn more about evidence on the impact of *i-Ready*, please visit CurriculumAssociates.com/Research.

@MyiReady Curriculum Associates @CurriculumAssoc iReady

Curriculum Associates